# ACHARYA NAGARJUNA UNIVERSITY::NAGARJUNA NAGAR-522 510
## DEPARTMENT OF MATHEMATICS
## M.Sc. Mathematics Program
## Curriculum and Syllabus
## (with effect from 2021 – 22 admitted batch)

## PROGRAM STRUCTURE
### Credits at a glance

| S.No. | Nature of the Course (S) | Credits |
|---|---|---|
| 1. | Core Courses | 64 |
| 2 | Electives | 16 |
| 3 | Moocs Courses | 08 |
| 4 | Project | 04 |
| 5 | Comprehensive viva voce | 04 |
| | Total number of credits | 96 |

6

### M.Sc. Mathematics Semester- I

| S.No. | Subject Code | Name of the Subject | Number of periods per week (Lectures/Seminar/ Tutorials) | credits | Assesment | |
|---|---|---|---|---|---|---|
| | | | | | Internal | End Semester |
| 1 | M101 | ALGEBRA | 06L+01S/T | 4 | 30% | 70% |
| 2 | M102 | ANALYSIS-I | 06L+01S/T | 4 | 30% | 70% |
| 3 | M103 | DIFFERENTIAL EQUATIONS | 06L+01S/T | 4 | 30% | 70% |
| 4 | M104 | TOPOLOGY | 06L+01S/T | 4 | 30% | 70% |
| 5 | M105 | ADVANCED DISCRETE MATHEMATICS | 06L+01S/T | 4 | 30% | 70% |
| Total Credits for Semester-I | | | | 20 | | |

1. B. Sah 29/9/2021

2. R. Srinu 29/09/2021

3. K. Gangadhar 29/9/21

4. G. Shobhalatha 29/9/2021

5. [illegible] 29/9/21.

6. [illegible] 29/9/21

7. P. [illegible] 29/9/21

B. Sah
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## M.Sc. Mathematics Semester-II

| S.No. | Subject Code | Name of the Subject | Number of periods per week (Lectures/Seminar/ Tutorials) | credits | Assesment | |
|---|---|---|---|---|---|---|
| | | | | | Internal | End Semester |
| 1 | M201 | GALOIS THEORY | 06L+01S/T | 4 | 30% | 70% |
| 2 | M202 | ANALYSIS-II | 06L+01S/T | 4 | 30% | 70% |
| 3 | M203 | MEASURE AND INTEGRATION | 06L+01S/T | 4 | 30% | 70% |
| 4 | M204 | NUMERICAL METHORS | 06L+01S/T | 4 | 30% | 70% |
| 5 | M205 | GRAPH THEORY | 06L+01S/T | 4 | 30% | 70% |
| | M2CV | MOOCS COURSE | | 4 | | |
| Total Credits for Semester-II | | | | 24 | | |

## M.Sc. Mathematics Semester-III

| S.No. | Subject Code | Name of the Subject | Number of periods per week (Lectures/Seminar/ Tutorials) | credits | Assesment | |
|---|---|---|---|---|---|---|
| | | | | | Internal | End Semester |
| 1 | M301 | RINGS AND MODULES | 06L+01S/T | 4 | 30% | 70% |
| 2 | M302 | COMPLEX ANALYSIS | 06L+01S/T | 4 | 30% | 70% |
| 3 | M303 | FUCNTIONAL ANALYSIS | 06L+01S/T | 4 | 30% | 70% |
| 4 Elective -I | M304(A) | FUZZY SETS AND THEIR APPLICATIONS | 06L+01S/T | 4 | 30% | 70% |
| | M304(B) | SEMI GROUPS | | | | |
| | M304(C) | NUMBER THEORY | | | | |
| 5 Elective -II | M305(A) | MATHEMATICAL BIOLOGY | 06L+01S/T | 4 | 30% | 70% |
| | M305(B) | LINEAR PROGRAMMING | | | | |
| | M305(C) | MATHEMATICAL MATHEDS | | | | |
| 6 | | MOOCS COURSE | | 4 | | |
| Total Credits for Semester-III | | | | 24 | | |

1. B. Sobhan 29/9/2021
2. R. Srinu 29/09/2021
3. K. Gangadhar 29/9/21
4. G. Shobhan 29/9/2021
5. T. Reddy 29/9/21
6. M. [illegible] 29/9/21
7. P. [illegible] 29/9/21

B. Sobhan
CHAIRMAN
P.G. Board of Studies in Mathematics
Acharya Nagarjuna University
Nagarjuna Nagar-522 510

## M.Sc. Mathematics Semester-IV

| S.No. | Subject Code | Name of the Subject | Number of periods per week (Lectures/Seminar/ Tutorials) | credits | Assesment | |
|---|---|---|---|---|---|---|
| | | | | | Internal | End Semester |
| 1 | M401 | NON-COMMUTATIVE RINGS | 06L+01S/T | 4 | 30% | 70% |
| 2 | M402 | PARTIAL DIFFERENTIAL EQUATIONS | 06L+01S/T | 4 | 30% | 70% |
| 3 | M403 | NEAR RINGS | 06L+01S/T | 4 | 30% | 70% |
| 4 Elective -III | M404(A) | ALGEBRAIC CODING THEORY | 06L+01S/T | 4 | 30% | 70% |
| | M404(B) | LATTICE THEORY | | | | |
| | M404(C) | OPERATOR THEORY | | | | |
| 5 Elective -IV | M405(A) | COMMUTATIVE ALGEBRA | 06L+01S/T | 4 | 30% | 70% |
| | M405(B) | BANACH ALGEBRA | | | | |
| | M405(C) | OPERATIONS RESEARCH | | | | |
| 6 | M4PRO | PROJECT | | 4 | | |
| 7 | M4PRV | Project VIVA VOCE | | 4 | | |
| Total Credits for Semester-IV | | | | 28 | | |

## Project:

The student will be given Project topics at the beginning of the IV semester by the faculty in-charge and the student has to present the topics, submit the hard copy of seminar to take report at the end of the IV semester. Out of a total of 100 marks, for the seminar evaluation, 50 marks for seminar report and record and 50 marks for the end semester examination (viva-voce). The Viva-Voce shall be conducted by a committee consisting of HOD, faculty in charge and a external examiner nominated by the university.

**Instructions for evaluation**

1. Each theory subject is evaluated for 100 Marks out of which 70 marks through end examination and internal assessment would be for 30 marks.
2. End Examination Question paper pattern is as follows:

1. B. Satyanarayana 29/9/2021

2. R. Srinivas 29/09/2021

3. G. Gangadhar 29/9/2021

4. G. Kishore 29/9/2021

5. T. Reddy 29/9/21.

6. 29/9/21

7. 29/9/21

B. Satyanarayana
CHAIRMAN
P.G. Board of Studies in Mathematics
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M.Sc DEGREE EXAMINATION, Month: ------- , Year ------- Paper Code:
Semester ----- , Mathematics
Paper No. ------- Name of the Subject --------
(with effect from ----- admitted bach)

Time: 3 hrs Maximum marks: 70

Answer any FIVE questions.One question from each unit. (5 x 14 marks = 70 marks)

UNIT-I

1(a)
(b)

(OR)

2(a)
(b)

UNIT-II

3(a)
(b)

(OR)

4(a)
(b)

UNIT-III

5(a)
(b)

(OR)

6(a)
(b)

UNIT-IV

7(a)
(b)

(OR)

8(a)
(b)

UNIT-V

9 (a)
(b)

(OR)

10.(a)
(b)

B. Sai
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## M.Sc. MATHEMATICS, I SEMESTER

## M101(21)(NR): ALGEBRA

## (With effect from the batch of students admitted during 2021-2022)

| | | | |
|---|---|---|---|
| Subject Code : | M101(21) | I A Marks | 30 |
| No. of Lecture / Seminar/ Tutorial for week | 06L + 01 S/T | End Exam Marks | 70 |
| | | Total Marks | 100 |

**Course Objectives:** To introduce the concepts Groups, Permutation groups, Direct product of groups, Rings, Euclidean Rings, Polynomial Rings and some related theories and to develop workingknowledge of these concepts and also skills for applying them in number theory and construction ofcertain fields.

### Unit-I

Group theory: Definition of a Group - Some Examples of Groups - Some Preliminary Lemmas - Subgroups - A Counting Principle - Normal Subgroups and Quotient Groups - Homomorphisms– Automorphisms.
(2.1 to 2.8 of the prescribed book [1])

**Learning outcomes**: Upon completion of this unit, the student will be able to: Understand theconcept of Groups, Normal groups and Quotients groups and automorphisms.

### Unit-II

Group Theory Continued: Cayley's theorem - Permutation groups-Another counting principle -Sylow's theorem.
( 2.9 to 2.12 of the prescribed book [1])

**Learning outcomes**: Upon completion of this unit, the student will be able to: Analyse Cayley's theorem, permutation groups, countingprinciple, Sylow's theorems and apply them for describing structures of finite groups.

### Unit-III

Direct products - finite abelian groups; Ring Theory: Definitions and Examples of Rings - some special classes of rings-Homomorphisms - Ideals and quotient Rings
(2.13 to 2.14 and 3.1 to 3.4 of the prescribed book [l])

**Learning outcomes**: Upon completion of this unit, the student will be able to: describe direct products, structures of finite abelian groups and demonstrate theknowledge of Rings, ideals of Rings and Quotient rings.

### Unit-IV

Ring Theory Continued: More Ideals and quotient Rings - The field of quotients of an Integral domain -Euclidean rings- A particular Euclidean ring -Polynomial Rings - Polynomials over the rational field.
(3.5 to 3.10 of the Prescribed book [l]).

R. Srinu

B. Sobi

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**Learning outcomes**: Upon completion of this unit, the student will be able to: DescribeField of Quotients of an integral domain, Euclidean rings,PolynomialRings and polynomial rings over the field of rational numbers.

## Unit-V

Polynomial Rings over Commutative Rings;Vector Spaces: Elementary Basic Concepts - Linear Independence and Bases - Dual spaces.
(3.11 and 4.1 to 4.3 of the prescribed book [1]).

**Learning outcomes**: Upon completion of this unit, the student will be able to: describe some other forms of polynomial rings and also base and dimension of a Vector Space and dual spaces.

**PRESCRIBED BOOK:** I I.N. Herstein, 'Topics in Algebra', Second Edition, John Wiley & Sons, 1999.

**REFERENCE BOOKS:**


1. P. B. Bhattacharya, S. K. Jain, S. R. Nagpaul. "Basic Abstract Algebra", Second Edition, Cambridge Press, 1995.
2. Thomas W. Hungerford, 'Algebra',Springer-Verlag, New York, 1974.
3. Serge Lang, 'Algebra', Revised Third Edition, Springer-Verlag, New York, 2002.


R. Srinu

B. Sat

CHAIRMAN
P.G. Board of Studies in Mathematics
Acharya Nagarjuna University
Nagarjuna Nagar-522 510.

CODE: M 1.1 (21)

# M.Sc. DEGREE EXAMINATION

First Semester

Mathematics

Paper I- ALGEBRA

MODEL PAPER

Time: Three hours | Maximum: 70 marks

Answer ONE question from each Unit. (5 x 14 = 70 marks)

## UNIT- I

1. (a) State and Prove Lagrange's theorem.

   (b) If $H$ and $K$ are finite subgroups of a group $G$ of orders $o(H)$ and $o(K)$ respectively, then prove that $o(HK)=\dfrac{o(H)o(K)}{o(H\cap K)}$.

**(OR)**

2. (a) State and prove Cauchy's theorem for abelian groups.

   (b) Prove that I(G) ≈G/Z, where I(G) is the group of inner automorphisms of the group G and Z is the center of G.

## UNIT- II

3. (a) State and Prove Cayley'stheorem.

   (b) Prove that if G is a finite group of order $p^2$ , p is a prime number , then G is abelian.

**(OR)**

4. (a) If $p$ is a prime number, $G$ is a finite group and $p^n / o(G)$ then prove that $G$ has a subgroup of order $p^n$ .

   (b) State and Prove the third part of the Sylow's theorem.

R Sri—

B. Sobhi

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## UNIT -III

**5.** (a) Show that if $G$ and $G^1$ are isomorphic abelian groups, then for every integers,s and $s^1$, $G(s)$ and $G(s^1)$ are isomorphic.

(b) Describe all finite abeliangroups of order (i) $2^6$ (ii) $2^3 \times 3^4$.

**(OR)**

6. (a) Prove that a finite integral domain is a field.

(b) If M is an ideal of the ring R, then prove that $R/M$ is a ring and is homomorphic image of R.

## UNIT- IV

7. (a) If $R$ is a commutative ring with unit element and $M$ is an ideal of $R$ then prove that $M$ is a maximal ideal of $R$ if and only if $R/M$ is a field.

(b) Let R be a Euclidean ring. Then prove that any two elements a and b in R have a greatest common divisor d and d = xa + yb for some x, y in R.

**(OR)**

8. (a) If $p$ is a prime number of the form $4n+1$ then prove that $p = a^2 + b^2$ for some integers $a$ and $b$.

(b) State and prove the Eisenstein Criterion.

## UNIT - V

9. (a)Prove that if R is an unique factorization domain then so is R[x].

(b) Prove that if V is a finite-dimensional vector space over F then any two bases of V have the same number of elements.

**(OR)**

10. (a) If $V$ is a finite dimensional vector space and $W$ is a subspace of $V$, then prove That $W$ is finite dimensional, $\dim W \le \dim V$ and $\dim V/W \le \dim V - \dim W$.

(b) Prove that if $V$ and $W$ are finite dimensional vector spaces of dimensions $m$ and $n$ respectively over $F$ then $Hom(V,W)$ is of dimension $mn$ over $F$.

R. Sri—

B. Sa—

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Acharya Nagarjuna University
Nagarjuna Nagar-522 510.

**DEPARTMENT OF MATHEMATICS** **M 102(21) (NR)**
**ACHARYA NAGARJUNA UNIVERSITY**
**M.Sc., Mathematics**
**SYLLABUS**
**(With effect from the batch of students admitted during 2021-2022)**

**I-SEMESTER**

**M 102-ANALYSIS-I**

| | | | |
|---|---|---|---|
| Subject Code: | M 102 | I A Marks | 30 |
| No. of Lecture / Seminar / Tutorial for week | 06L + 01 S/T | End Exam Marks | 70 |
| | | Total Marks | 100 |

**Course Objectives:** The course objective is to develop problem solving skills and to acquire knowledge on some of the basic concepts in numerical sequences, series, limits, derivatives, and Riemann Stieltjes –integrals..

**UNIT-I**

Numerical Sequences and Series: Convergent sequences, Subsequences, Cauchy Sequences.
(3.1 to 3.14 of Chapter 3 of the Text book) **(Questions not to be given in 3.1 to 3.14)**

Upper and Lower limits, Some special sequences, Series, Series of non-negative terms , Number e , The Root and Ratio tests, Power series , Summation by parts , Absolute convergence , Addition and Multiplication of series.
(3.15 to 3.51 of Chapter 3 of the Text book)

**Learning Outcomes:**
Upon completion of this unit, the student will be able to: Understand the concepts of numerical sequences, series, and limits. Compute the limits of some sequences and `e' with great accuracy. Become familiar with a number of series of nonnegative terms whose convergence or divergence ~~iwn.~~ is known.

**UNIT-II**

Continuity: Limits of functions, Continuous functions, Continuity and Compactness, Continuity and Connectedness. Discontinuities, Monotonic functions, Infinite limits and limits at infinity.
(Chapter 4 of the Text book)

**Learning Outcomes:** Upon completion of this unit, the student will be able to: Understand the concepts of limit and continuity of functions and discuss types of discontinuities.

**UNIT-III**

Differentiation: Derivative of a real function ,Mean value theorems, The continuity of derivatives, L'Hospital's rule, Derivatives of higher order, Taylor's theorem.
(5.1 to 5.15 of Chapter 5 of the Text book)

**Learning Outcomes:** Upon completion of this unit, the student will be able to:Study another equally important concept namely differentiation that is essential in the study of velocity and acceleration of continuous paths.

KPrasad

B. Sobha

CHAIRMAN
P.G. Board of Studies in Mathematics
Acharya Nagarjuna University
Nagarjuna Nagar-522 510.

**UNIT-IV**

Differentiation of vector-valued functions.

Riemann-Stieltjes Integral: Definition and Existence of the Integral.

(5.16 to 5.19 of Chapter 5 and 6.1 to 6.11 of Chapter 6 of the Text book)

**Learning Outcomes:** Upon completion of this unit, the student will be able to: Determine the Riemann – Stieltjes integrability of a bounded function and prove a selection of theorems concerning integration.

**UNIT-V**

Properties of the Integral, Integration and Differentiation, Integration of vector-valued functions , Rectifiable curves.

(6.12 to 6.27 of Chapter 6 of the Text book)

**Learning Outcomes:** Upon completion of this unit, the student will be able to: Prove integration and differentiation are (in a certain sense) inverse operations and prove a selection of theorems concerning integration

**TEXT BOOK:**

Principles of Mathematical analysis by Walter Rudin 3rd Edition.

**REFERENCE BOOK:**

Mathematical Analysis by Tom M. Apostal, Narosa Publishing House, 2nd Edition, 1985.

**Course Outcomes:** After completing this course, the student gets adequate knowledge on numerical sequences and series and also about the behaviour of a function in the vicinity of a point, learns about discontinuities at a point, analytical study of the moment of particle in the plane as well as the areas of the region bounded by a curve and the axes.

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**CODE: M 1.2(21) (NR)**

M.Sc. DEGREE EXAMINATION, OCTOBER/NOVEMBER 2021

First Semester

Mathematics

Paper II - ANALYSIS – I (MODEL PAPER)

Time: Three hours | Maximum : 70 marks

Answer ONE question from each Unit. (5 × 14 = 70 Marks)

**UNIT I**

1. (a) Prove that $\lim_{n\to\infty}\frac{n^{\alpha}}{(1+p)^{n}}=0$, where $p>0$ and $\alpha$ is real.

(b) Suppose $a_1 \ge a_2 \ge a_3 \ge ...... \ge 0$. Then the series $\sum_{n=1}^{\infty} a_n$ converges if and only if the series $\sum_{k=0}^{\infty} 2^k a_{2^k}$ converges.

**(OR)**

2. (a) For any sequence $\{c_n\}$ of positive numbers, show that $\limsup_{n\to\infty}\sqrt[n]{c_n} \le \limsup_{n\to\infty}\frac{c_{n+1}}{c_n}$.

(b) Suppose (i) $\sum_{n=0}^{\infty} a_n$ converges absolutely,

(ii) $\sum_{n=0}^{\infty} a_n = A$,

(iii) $\sum_{n=0}^{\infty} b_n = B$.

Then show that $\sum_{n=0}^{\infty} c_n$ converges to $AB$, where $c_n = \sum_{k=0}^{n} a_k b_{n-k}, n = 0,1,2,......$

**UNIT II**

3 (a) Prove that a mapping $f$ of a metric space $X$ into a metric space $Y$ is continuous on $X$ If and only if $f^{-1}(V)$ is open in $X$ for every open set $V$ in $Y$.

(b) Suppose $f$ is a continuous mapping of a compact metric space $X$ into a metric space $Y$. Then prove that $f(X)$ is compact.

**(OR)**

4 (a) Let $f$ be a continuous mapping of a compact metric space $X$ into a metric space $Y$. Then prove that $f$ is uniformly continuous on $X$.

(b) Let $f$ be monotonic on $(a,b)$. Then prove that the set of points of $(a,b)$ at which $f$ is discontinuous is at most countable.

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## UNIT III

5 (a) Let $f$ and $g$ be continuous real functions on $[a,b]$ which are differentiable in $(a,b)$. Then prove that there is a point $x \in (a,b)$ at which

$[f(b)-f(a)]g^1(x)=[g(b)-g(a)]f^1(x).$

(b) If $c_0+\frac{c_1}{2}+.........+\frac{c_{n-1}}{n}+\frac{c_n}{n+1}=0$, where $c_0,c_1,c_2,.........,c_n$ are real constants, then prove that the equation $c_0+c_1x+........+c_{n-1}x^{n-1}+c_nx^n=0$ has atleast one real root between 0 and 1.

**(OR)**

6 (a) Suppose $f$ is a real differentiable function on $[a,b]$ and suppose $f^1(a)<\lambda<f^1(b)$. Then prove that there is a point $x\in(a,b)$ such that $f^1(x)=\lambda$.

(b) State and prove Taylor's theorem.

## UNIT IV

7 (a) Suppose $\overline{f}$ is a continuous mapping of $[a,b]$ into $\mathbb{R}^k$ and $\overline{f}$ is differentiable in $(a,b)$.

Then prove that there exists $x\in(a,b)$ such that $\left|\overline{f}(b)-\overline{f}(a)\right|\le(b-a)\left|\overline{f}(x)\right|$.

(b) State and prove the necessary and sufficient condition for the existence of Riemann-Stieltjes integral.

**(OR)**

8 (a) Suppose $f$ is bounded on $[a,b]$, $f$ has only finitely many points of discontinuity on $[a,b]$, and $\alpha$ is continuous at every point at which $f$ is discontinuous. Then prove that $f\in\Re(\alpha)$ on $[a,b]$.

(b) Suppose $f\in\Re(\alpha)$ on $[a,b]$, $m\le f\le M$, $\varphi$ is continuous on $[m,M]$, and $h(x)=\varphi(f(x))$ on $[a,b]$. Then prove that $h\in\Re(\alpha)$ on $[a,b]$.

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## UNIT V

9 (a) If $f \in \Re(\alpha)$ and $g \in \Re(\alpha)$ on $[a,b]$, then prove that (i) $fg \in \Re(\alpha)$ and

(ii) $|f| \in \Re(\alpha)$ and $\left|\int_a^b f\, d\alpha\right| \leq \int_a^b |f|\, d\alpha.$

(b) Assume $\alpha$ is increasing and $\alpha^1$ is Reimann integrable on $[a,b]$. For any bounded function $f$ defined on $[a,b]$, prove that $f \in \Re(\alpha)$ if and only if $f\alpha^1$ is Reimann integrable.

**(OR)**

10 (a) State and prove the fundamental theorem of calculus.

(b) If $\gamma^1$ is continuous on $[a,b]$, then show that $\gamma$ is rectifiable and that

$$\wedge(\gamma) = \int_a^b |\gamma^1(t)|\, dt.$$

B. Sobhi
CHAIRMAN
P.G. Board of Studies in Mathematics
Acharya Nagarjuna University
Nagarjuna Nagar-522 510.

**M.Sc. MATHEMATICS, I SEMESTER**
**M103(21)(NR): DIFFERENTIAL EQUATIONS~~ALGEBRA~~**
**(With effect from the batch of students admitted during 2021-2022)**

| Subject Code : | M103(21) | I A Marks | 30 |
|---|---|---|---|
| No. of Lecture / Seminar/ Tutorial for week | 06L + 01 S/T | End Exam Marks | 70 |
| | | Total Marks | 100 |

**Course Objectives:** To provide some standard methods for solving first-order, second-order and higher-order homogeneous and nonhomogeneous ordinary differential equations with constant and variable coefficients, linear equation with regular singular points, and to study the method of successive approximations, Lipshitz condition and non-local existence of solutions.

**UNIT-I: Linear equations of the first order**: Linear equations of the first order – The equation $Y^1 + ay = 0$ – The equation $y^1 + ay = b(x)$ - The general linear equation of the first order. (Sections 4-7 Chapter 1 of Prescribed Text book).

**Linear Equations with constant co-efficients:** Introduction - The second order Homogeneous equation – Initial value problems for the second order equations. (Sections 1 to 3 in Chapter 2 Prescribed Book).

**Learning outcomes**: Upon completion of this unit, the student will be able to: Obtain the solutions of first order linear differential equations, second order homogeneous equations and initial value problems for the second order equations.

**UNIT – II**

**Linear Equations with constant co-efficients:** Linear dependence and independence – A formula for the Wronskian – The non-homogeneous equation of order two – The homogeneous equation of order n – Initial value problems for n-th order equations.(Sections 4 to 8 in Chapter 2 Prescribed Text Book).

**Learning outcomes**: Upon completion of this unit, the student will be able to: Obtain the solutions of non-homogeneous linear differential equations with constant coefficients and understand the utility of Wronskian, linear independence and independence of solutions.

**UNIT – III: Linear Equations with Variable Co-efficients**: Introduction – Initial value problems for the homogeneous equation – Solutions of the homogeneous equation – The Wronskian and linear independence – Reduction of the order of a homogeneous equation – The non-homogeneous equation – Homogeneous equations with analytic coefficients. (Sections 1 to7 in Chapter 3 Prescribed Text Book).

**Learning outcomes**: Upon completion of this unit, the student will be able to: learn how to solve homogeneous and non-homogeneous differential equations with variable coefficients and homogeneous equation with analytic co-efficient.

**UNIT – IV : Linear Equations with Regular Singular Points:** Introduction – The Euler equation – Second order equations with regular singular points – A convergence proof - The exceptional cases – The Bessel equation.(Sections 1 to 7 in Chapter 4 Prescribed Text Book).

**Learning outcomes**: Upon completion of this unit, the student will be able to: Understand the concepts regular singular points and solve the Euler equation and the Bessel equation.

**UNIT- V: Existence and Uniqueness of Solutions to First Order Equations:** Introduction – Equation with variables separated – Exact equations – The method of successive approximations – The Lipschitz condition – Convergence of the successive approximations – Non-local existence of solutions. ( Sections 1 to 7 in Chapter 5 Prescribed Text Book).

B. Sa[illegible]
CHAIRMAN
P.G. Board of Studies in Mathematics
Acharya Nagarjuna University
Nagarjuna Nagar-522 510.

**Learning outcomes**: Upon completion of this unit, the student will be able to: Understand the concepts of successive approximations, The Lipschitz condition and prove local and Non-local existence theorems.

**Prescribed Text Book** : An introduction to Ordinary Differential Equations by Earl A. Coddington, Prentice-hall of Indial Private Limited, NEW DELHI, 1974.

**Course outcomes**: The students shall receive good introduction to the study of solutions of equations in higher order derivatives of a variable function with variable coefficients in general and constant coefficients as well as the student also learns technique of finding solutions of somespecial types of equations. Finally the student learns how to establish existence and uniqueness of $y^1 = f(x, y)$ when $f$ satisfies the Lipschitz condition.

B. S[illegible]

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P.G. Board of Studies in Mathematics
Acharya Nagarjuna University
Nagarjuna Nagar-522 510.

CODE: M103(21)(NR)

# M.Sc. DEGREE EXAMINATION, MARCH 202__

## FIRST SEMESTER

## MATHEMATICS

## Paper –III, DIFFERENTIAL EQUATIONS

Model paper

Time: Three hours Maximum : 70 marks

Answer ONE question from each Unit (5 x 14 = 70 marks)

UNIT-I

1. (a) Find all solutions of the equation $y^1 + 2xy = x$
   (b) Find all solutions $\emptyset$ $of$ $y^{11}+y = 0$ satisfying $\emptyset(0) = 1, \emptyset(\pi/2)=2$.

(OR)

2 a) let $\propto, \beta$ be any two constants, and let $x_o$ be any real number. On any interval I containing $x_o$ there exists at most one solution $\emptyset$ of the initial value problem. L(y)=0 , $y(x_0) =\propto, y^1(x_0) =\beta$

UNIT-II

3. (a) Prove that two solutions $\emptyset_1\emptyset_2$ of L (y) = 0 are linear independent on an interval I if , and only if, W $(\emptyset_1\emptyset_2)$ (x)$\neq$0
   (b) Let $\emptyset_1\emptyset_2$ be two differentiable functions on an interval I, which are not necessarily solutions of an equation L(y) = 0. Prove that If $\emptyset_1\emptyset_2$ are linear independent on an interval I then W $(\emptyset_1\emptyset_2)$ (x)=0 for all x in I.

(OR)

4.(a) Find all solutions of the equation $y^{11}+9y = \sin 3x$ .
(b) Compute that solution $\emptyset$ of this equation $y^{(4)}+16y=0$ which satisfies
$\emptyset(0) = 1$ , $\emptyset^1(0) =0$ , $\emptyset^{11}(0)=0$ , $\emptyset^{111}(0) = 0$.

UNIT-III

5.(a) One solution of $x^3y^{111} - 3x^2y^{11} + 6xy^1 - 6y = 0$
for x >0 is $\emptyset_1(x) = x$. Find a basis for the solutions for x >0.
(b) One solution of $x^2y^{11} - 2y = 0$ on $0<x<\propto$ is $\emptyset(x) = x^2$. Find all solutions of
$x^2y^{11} - 2y = 2x - 1$ on $0<x<\propto$ .

(OR)

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6.(a) State and prove Existence Theorem for Analytic Coefficients.

(b) Find two linear independent power series solutions (in powers of x) of the following equation $y^{11}-xy^{1}+y=0$

UNIT-IV

7. (a) Find all solutions $\emptyset$ of the form

$\emptyset(x) = |x|^r \sum_{k=0}^{\propto} c_{kx^k}$, $(|x| > 0)$ for the equation $3x^2y^{11}+5xy^1+3xy=0$.

(b) Obtain two linear independent solutions of the following equation which are valid near x=0

$x^2y^{11}+3xy^1+(1+x)y=0$.

(OR)

8. Prove that the series defining $J_o$ and $k_o$ coverage for $|x| < \propto$

UNIT-V

9.(a) Let M,N be two real -valued functions which have continuous first partial derivatives on some rectangle R. $|x-x_o| \leq a$, $|y-y_o| \leq b$ then the equation $M(x,y)\,dx + N(x,y)\,dy = 0$ is exact in R if,and only if, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ in R.

(b) The equations below are written in the form $M(x,y)\,dx + N(x,y)\,dy = 0$, where M, N exist on the whole plane . Determine which equation is exact there, and solve

$2xy\ dx + (x^2+3y^2)\ dy = 0$

(OR)

10. State and Prove Non-local Existence Theorem.

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**M.Sc. MATHEMATICS I SEMESTER**
**M104(21)(NR): TOPOLOGY**
**(With effect from the batch of students admitted during 2021-2022)**

| | | | |
|---|---|---|---|
| Subject Code : | M104(21) | I A Marks | 30 |
| No. of Lecture / Seminar/Tutorial for week | 06L + 01 S/T | End Exam Marks | 70 |
| | | Total Marks | 100 |

**Course Objectives:** To generalize the concept of distance, open sets, closed sets and related theorems in real line and to learn basic concepts in Metric Spaces, Topological Spaces, compact spaces and connected spaces.

**UNIT-I**

**Metric Spaces:** Definition and some examples, Open sets, Closed sets, Convergence, completeness and Baire's theorem, Continuous mappings. (Sections 9 to 13 of chapter 2)

**Learning outcomes**: Upon completion of this unit, the student will be able to: Understand the basic concepts of metric spaces, open sets, closed sets and continuous functions on metric spaces.

**UNIT-II**

**Topological spaces**: The Definition and some examples, Elementary Concepts, Open bases and open subbases, Weak topologies. (Sections 16 to 19 of chapter 3)

**Learning outcomes**: Upon completion of this unit, the student will be able to: Define and illustrate the concept of topology and prove a selection of theorems concerning Topological spaces, continuous functions and product topologies.

**UNIT-III**

**Compactness**: Compact spaces, Products of spaces, Tychonoff's theorem and locally compact spaces, Compactness for metric spaces, Ascoli's theorem. (Sections 21 to 25 of chapter 4)

**Learning outcomes**: Upon completion of this unit, the student will be able to: Characterize compact spaces using the Heiene-Borel theorem.

**UNIT-IV**

**Separation**: $T_1$ –spaces and Hausdorff spaces, completely regular spaces and normal spaces, Urysohn's Lemma and the Tietze extension theorem. (Sections 26 to 28 of chapter 5).

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**Learning outcomes**: Upon completion of this unit, the student will be able to: Define and illustrate the concepts of the separation axioms and appreciate the beauty of deep mathematical results like Tietze Extension theorem.

**UNIT-V**

The Urysohnimbedding theorem, Connected spaces, The components of a space (Section 29 of chapter 5 and sections 31 to 32 of chapter 6).

**Learning outcomes**: Upon completion of this unit, the student will be able to: Define and illustrate the concepts of the mathematical results like Urysohn's lemma, Urysohn imbedding theorem and understand the dynamics of the proof techniques. Characterize connected spaces, components of a space.

**TEXT BOOK:**

Introduction to Topology and Modern Analysis by **G.F. Simmons**, McGraw-Hill Book Company, New York International student edition.

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CODE: M 1.4(21)(NR)

# M.Sc. DEGREE EXAMINATION

## First Semester

## Mathematics

## Paper IV- TOPOLOGY

## MODEL PAPER

Time: Three hours Maximum: 70 marks

Answer ONE question from each Unit. (5 x 14 = 70 marks)

### UNIT I

1. (a) Let X be a Metric space. Then show that a subset F of X is closed iff its complement $F'$ is Open.
   (b) State and prove Cantor intersection theorem.

OR

2. (a) Let X be a Metric space. If $\{x_n\}$ and $\{y_n\}$ are sequences in X such that $x_n \longrightarrow x$ and $y_n \longrightarrow y$ then show that $d(x_n, y_n) \longrightarrow d(x,y)$.
   (b) Let X and Y be metric spaces and f be a mapping of X in to Y. Then show that f is continuous if and only if $x_n \longrightarrow x_0$ and $f(x_n) \longrightarrow f(x_0)$

### UNIT II

3. (a) Let X be a non-empty set, and let there be given a class of subsets of X, which is closed under the formation of arbitrary intersections and finite unions. Then prove that the class of these sets is a topology on X whose close sets are precisely those initially given.

   (b) Let X be a second countable space. Then prove that any open base for X has a countable subclass which is also an open base.

OR

4. (a) Let X be a non-empty set , and let $S$ be an arbitrary class of subsets of X .Then prove that $S$ can serve as an open sub base for a topology on X.
   (b) Let X be a non empty set. Then prove that the family of all topologies on X is a complete Lattice with respect to the relation "is weaker than".

### UNIT III

5. (a) Prove that a topological space is compact if and only if every class of closed sets with empty intersection has a finite subclass with empty intersection.

   (b) State and prove Tychonoff's theorem.

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OR

6. (a) Prove that every compact metric space has the Bolzano –Weierstrass property.

   (b) Show that a metric space is compact iff it is complete and totally bounded

**UNIT IV**

7. (a) **Show** that every compact subspace of a Hausdorff space is closed.
   (b) Show that every compact Hausdorff space is normal.

OR

8. State and prove Tietze extension theorem.

**UNIT V**

9. (a) Prove that the spaces $R^n$ and $C^n$ are connected.

   (b) Prove that any non -empty class of connected spaces is connected.

OR

10. State and prove Urysohn Imbedding theorem.

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# M.Sc. MATHEMATICS I SEMESTER
# M105(21)(NR): ADVANCED DISCRETE MATHEMATICS
## (With effect from the batch of students admitted during 2021-2022)

| | | | |
|---|---|---|---|
| Subject Code : | M105(21) | I A Marks | 30 |
| No. of Lecture / Seminar/ Tutorial for week | 06L + 01 S/T | End Exam Marks | 70 |
| | | Total Marks | 100 |

**Course Objectives :**To develop skills and to acquire knowledge on some of the basic concepts in Truth tables, Tautology and contradiction, Tautological implication, variables and quantifiers, Logic, Finite Machines, Fundamental concepts and basic results of Boolean Algebra, Lattices and their Applications, and applications of switching circuits, Gatting Network, Karnag diagrams..

**UNIT –I:** Propositional Calculus: Statements and Notations- Connectives and Truth Tables – Tautology and Contradiction – Equivalence of Statement / Formulas – Duality Law and Tautological Implication – Normal Forms . (Chapter – I of the reference [3]).

**Learning outcomes**: Upon completion of this unit, the student will be able to: Formulate statements from common language to formal logic, apply truth tables and Normal Forms.

**UNIT –II:** The theory of Inference for Statement Calculus – Consistency of Premises and Indirect Method of Proof. (Chapter – I of the reference [3]).
Predicate Calculus: Predicate Logic – Statement Functions, Variables and Quantifiers – Free and Bound Variable – Inference Theory for the Predicate Calculus (Chapter – 2 of the reference [3]).

**Learning outcomes**: Upon completion of this unit, the student will be able to: Understand the rules of propositional and predicatecalculus.

**UNIT –III: Finite Machines :** Introduction, state tables and state diagrams, simple properties, Dynamics and Behavior. (refer Chapter 5 of the reference book [1]).

**Learning outcomes**: Upon completion of this unit, the student will be able to: Understand the concept of finite machines and study their applications like minimization, and realization.

**UNIT – IV:** Properties and Examples of Lattices, Distributive Lattices, Boolean polynomials. (Sections 1 to 4 of Chapter 1 of [2] ).

**Learning outcomes**: Upon completion of this unit, the student will be able to: be familiar with the notions of ordered algebraic structures, including lattices and Boolean algebras.

**UNIT –V:** Ideals , filters and equations, Minimal forms of Boolean polynomials, Application of Lattices: Application of switching circuits, ( Sections 5,6 of Chapter -1 and sections 7 and 8 of Chapter 2of [2]).

**Learning outcomes**: Upon completion of this unit, the student will be able to: Understand the concept of Boolean polynomials, ideals, filters and calculate the minimal forms of Boolean polynomials. Demonstrate switching circuits and applications of switching circuits.

Note: For units –III and IV the material of pages 1 to 66 of [2] is to be covered.

**REFERENCE BOOKS:**

[1] "Application oriented Algebra" JAMES L FISHER , IEP, Dun- Downplay pub.1977.
[2] " Applied abstract algebra", Second Edition, R.LIDL AND G.PILZ, Springer,1998.
[3] " Bhavanari Satyanarayana, Tumurukota Venkata Pradeep Kumar and Shaik Mohnddin Shaw, "Mathematical Foundation of Computer Science" BS Publications (A unit of BSP Book Pvt Ltd), Hyderabad, India 2016. (ISBN. 978-93-83635-81-8).
[4] Rm. Somasundaram "Discrete Mathematical Structures" Prentice Hall of India, 2003.

[5] Bhavanari Satyanarayana & Kuncham Syam Prasad, "Discrete Mathematics and Graph theory"(For B.Tech/B.Sc./M.Sc (Maths)), Printice Hall of India, New Delhi, April 2014.

**Course Outcomes:** After competing this course, the student will be able to: Receive meaningfulintroduction to discrete mathematics and its applications.

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CODE: M105(12)(NR)

# M.Sc. DEGREE EXAMINATION, MARCH 202__

## FIRST SEMESTER

## MATHEMATICS

## Paper –V, ADVANCED DISCRETE MATHEMATICS

Model paper

Time: Three hours Maximum : 70 marks

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Answer ONE question from each Unit (5 x 14 = 70 marks)

### UNIT _I

1 a) Show the implication $[(p\rightarrow q)\rightarrow q]\Rightarrow(p \vee q)$.

b) Show that the proposition $[(p \vee \neg q)\wedge(\neg p \vee \neg q)] \vee q$ is a Tautology.

(OR)

2 a) Construct the truth tables of converse ,inverse and contrapositive of the proposition $p\rightarrow q$.

b) Define PDNF. Find PDNF for $(\neg x \vee y)$

### UNIT-II

3 a) Show that the conclusions C: $\neg P$ follows from the premises $H_1$: $\neg P \vee Q$, $H_2$: $\neg(Q \wedge \neg R)$ and $H_3$: $\neg R$.

b)"If there was a partythen catching the train was difficult.If they arrived on timethen catching the train was not difficult.They arrived on time.therfore there was no party".Show that the statement constitutes a valid argument.

(OR)

4 a) Define Quantifiers.Symbolyse "All the people respects selfless leaders".

b) Using proof by contradiction show that $\sqrt{2}$ is not a rational number.

### UNIT-III

5 a) Let $\mathscr{P}$ be a state machine congruence on $M=(\mathscr{S},\mathscr{I},\delta)$. Then show that there is a state homomorphism f from M onto $\overline{M}=(\mathscr{P},\mathscr{I},\bar{\delta})$ given by f(s)=[s].

b) Let f be a state homomorphism from the state machine $M=(\mathscr{S},\mathscr{I},\delta)$ onto the state machine $M_1=(\mathscr{S}_1, \mathscr{I}, \delta_1)$. Then show that there is a state machine congruence on M such that $\overline{M}$ is isomorphic to $M_1$.

(OR)

6 a) Minimize the number of states for the machine given by the following state table

| | $\delta$ | | $\theta$ | |
|---|---|---|---|---|
| | 0 | 1 | 0 | 1 |
| $S_0$ | $S_0$ | $S_2$ | 0 | 0 |
| $S_1$ | $S_2$ | $S_5$ | 1 | 0 |
| $S_2$ | $S_2$ | $S_2$ | 1 | 1 |
| $S_3$ | $S_1$ | $S_1$ | 1 | 1 |
| $S_4$ | $S_2$ | $S_3$ | 0 | 1 |
| $S_5$ | $S_4$ | $S_5$ | 1 | 1 |
| $S_6$ | $S_2$ | $S_6$ | 1 | 1 |

## UNIT-IV

7 a) Prove that a lattice is distributive iff it does not contain a sublattice isomorphic to the diamond lattice.

b) Define a modular lattice . Show that every distributive lattice is modular.

(OR)

8 a) Let $(L, \leq)$ be a lattice ordered set . Define $x \wedge y = \text{Inf}(x, y)$ and $x \vee y = \text{Sup}(x, y)$, $x, y \in L$. Then Show that $(L, \wedge, \vee)$ is an algebraic lattice.

b) Find the D N F of $x_1(x_2+x_3)' + (x_1x_2+x_3')x_1$

## UNIT-V

9 a) Let B be a Boolean algebra. Show that an ideal M in B is maximal iff for any $b \in M$ (or) $b' \in M$ but not both hold.

b) Show that a polynomial $p \in P_n$ is equivalent to the sum of all prime implicants of p.

(OR)

10 a) Determine the prime implicants of the Boolean Poilnomial $Z = x_1x_2x_3x_4' + x_1x_2x_3'x_4' + x_1x_2'x_3x_4 + x_1x_2'x_3x_4' + x_1'x_2'x_3x_4 + x_1'x_2'x_3x_4' + x_1'x_2'x_3'x_4$. Hence find out the minimal form using the Quine-Mc Clusky Method.

B. S—
CHAIRMAN
P.G. Board of Studies in Mathematics
Acharya Nagarjuna University
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**M.Sc. MATHEMATICS, I I SEMESTER**
**M201(21)(NR): GALOIS THEORY**
**(With effect from the batch of students admitted during 2021-2022)**

| | | | |
|---|---|---|---|
| Subject Code : | M201(21) | I A Marks | 30 |
| No. of Lecture / Seminar/ Tutorial for week | 06L + 01 S/T | End Exam Marks | 70 |
| | | Total Marks | 100 |

**Course Objectives:** To develop skills and to acquire knowledge on some of the basic concepts in, Algebraic Extensions, Splitting fields, normal and separable extensions, fundamental theorem of Galois theory, fundamental theorem of algebra and applications of Galois theory to classical problems.

**UNIT-I**

Algebraic extensions of fields: Irreducible polynomials and Eisenstein criterion- Adjunction of roots - Algebraic extensions.
(Sections 15.1 to 15. 3 of Chapter15 of the Prescribed book)

**Learning outcomes**: Upon completion of this unit, the student will be able to: Derive and apply Gauss Lemma, Eisenstein criterion for irreducibility of Polynomials over the field of rational numbers and algebraic extensions.

**UNIT-II**

Algebraically closed fields;Normal and Separable extensions: Splitting fields - Normal extensions -Multiple roots.
(Section 15.4 of Chapter 15 and Sections 16.1 to 16.3 of Chapter16of the prescribed book)

**Learning outcomes**: Upon completion of this unit, the student will be able to:
Demonstratealgebraically closed fields, splitting fields, normal extensions and multiple roots.

UNIT-III

Finite fields - Separable extensions-Automorphism groups and fixed fields.
(Sections 16.4 to 16.5 of Chapter 16and Section 17.1 of Chapter 17 of the prescribed book)

**Learning outcome**: Upon completion of this unit, the student will be able to: Learn and apply finite fields, separable extensions and fixed fields of automorphism groups.

UNIT-IV

Galois Theory: Fundamental theorem of Galois theory - Fundamental theorem of Algebra; Applications of Galois theory to classical problems: Roots of unity and cyclotomic polynomials - Cyclic extensions.

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(Sections 17.2 to 17.3 of Chapter 17 and Sections 18.1 to 18.2 of Chapter 18 of the prescribed book)

**Learning outcomes**: Upon completion of this unit, the student will be able to: Understand the fundamental theorem of Galois theory, the fundamental theorem of algebra, roots of unity and cyclotomic polynomials and cyclic extensions.

UNIT-V

Polynomials solvable by radicals -Symmetric functions – Ruler and Compass constructions

(Sections 18.3 to 18.5 of Chapter 18 of the prescribed text book)

**Learning outcomes**: Upon completion of this unit, the student will be able to: Understand polynomials solvable by radicals, symmetric functions and Ruler & Compass constructions.

**PRISCRIBED BOOK**:

P. B. Bhattacharya, S. K. Jain, S. R. Nagpaul. "Basic Abstract Algebra", Second Edition, Cambridge Press, 1995.

**REFERENCE BOOKS:**

1. I.N. Herstein, 'Topics in Algebra', Second Edition, John Wiley &Sons, 1999.
2. Thomas W. Hungerford , 'Algebra', Springer-Verlag, New York, 1974.
3. Serge Lang, 'Algebra', Revised Third Edition, Springer-Verlag, New York, 2002.

R. Srinivasan

B. Satyanarayana

CHAIRMAN
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Nagarjuna Nagar-522 510.

CODE: M 201 (21)(NR)

M.Sc. DEGREE EXAMINATION
SECOND SEMESTER
MATHEMATICS
Paper I– GALOIS THEORY
MODEL PAPER

Time : Three hours | Maximum : 70 marks

Answer ONE question from each Unit. $(5 \times 14 = 70$ Marks)

UNIT I

1. (a) Let $F \subseteq E \subseteq F$ be fields. If $[K: E] < \infty$ and $[E: F] < \infty$, then show that

(i) $[K: F] < \infty$

(ii) $[K: F] = [K: E]\,[E: F]$.

(b) Let p (x) be an irreducible polynomial in F [x]. Then show that there exists an extension E of F in which p (x) has a root.

**(OR)**

2. (a) Let p (x) be an irreducible polynomial of degree n in F [x] and let u be a root of p(x) in an extension E of F. Then prove that $[F(u) : F] = n$.

(b) If E is an algebraic extension of F and $\tau: E \rightarrow E$ is an embedding of E into itself over F, then show that $\tau$ is onto E.

UNIT II

3. Let F be a field. Define an algebraic closure of F and show that F has an algebraic closure.

**(OR)**

4.(a) Let F be a field and K and E be splitting fields of $f(x) \in F[x]$ over F. Then prove that there is an isomorphism of K onto E which is identity on F.

(b) If $f(x) \in F[x]$ is irreducible over F, then show that all roots of $f(x)$ have the same multiplicity.

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B. Satyanarayana

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UNIT III

5. (a) Show that the prime field of a field F is either isomorphic to Q or to $Z/(p)$, p is a prime.

(b) Show that any two finite fields with $p^n$ elements are isomorphic.

**(OR)**

6. (a) Show that a finite separable extension of a field F is a simple extension of F.

(b) State and prove Dedekind lemma.

UNIT IV

7. State and prove fundamental theorem of Galois theory.

**(OR)**

8. Show that $\Phi_n(x) = \prod(x - \omega)$, $\omega$ primitive nth root of unity in the field of complex numbers C, is an irreducible polynomial of degree $\phi(n)$ in Z[x].

UNIT V

9. (a) Show that $f(x) \in F[x]$ is solvable by radicals over F if and only if its splitting field E over F has solvable group $G(E / F)$.

(b) Show that the polynomial $2x^5 - 5x^4 + 5$ is not solvable by radicals over Q.

**(OR)**

10. (a) If a positive real number a is constructable, then show that $\sqrt{a}$ is also constructable.

(b) Show that there exists an angle that can't be trisected by using ruler and compass alone.

R. Srinu

B. Sobha

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**DEPARTMENT OF MATHEMATICS** **M 202(21) (NR)**
**ACHARYA NAGARJUNA UNIVERSITY**
**M.Sc., Mathematics**
**SYLLABUS**
**(With effect from the batch of students admitted during 2021-2022)**

## II-SEMESTER

## M 202 –ANALYSIS-II

| Subject Code: | M 202 | I A Marks | 30 |
|---|---|---|---|
| No. of Lecture / Seminar / Tutorial for week | 06L + 01 S/T | End Exam Marks | 70 |
| | | Total Marks | 100 |

**Course Objectives:** To introduce the concepts of sequences and series of functions, equicontinuous family of functions, power series, linear transformations, contraction principle, derivatives of higher order, differentiation of integrals and to prove the inverse function theorem and implicit function theorem.

### UNIT-I

Sequences and series of functions: Discussion of main problem, Uniform convergence, Uniform convergence and Continuity, Uniform convergence and Integration.
(7.1 to 7.16 of Chapter 7 of the Text Book)

**Learning Outcomes:** Upon completion of this unit, the student will be able to:
Recognize the difference between point wise and uniform convergence of sequences of functions and illustrate the effect of uniform convergence on the limit function with respect to continuity, and integrability.

### UNIT-II

Uniform Convergence and Differentiation, Equicontinuous families of functions, Stone-Weierstrass theorem.
(7.17 to 7.27 of Chapter 7 of the Text Book)

**Learning Outcomes:** Upon completion of this unit, the student will be able to: Illustrate the effect of uniform convergence on the limit function with respect to differentiability. Study the Stone – Weierstrass theorem and its applications.

### UNIT-III

Algebra of functions, Power series, Exponential and logarithmic functions, Trigonometric functions.
(7.28 to 7.33 of Chapter 7 and 8.1 to 8.7 of Chapter 8 of the Text Book)

**Learning Outcomes:** Upon completion of this unit, the student will be able to:
Understand the properties of power series. Study the Exponential, Logarithmic and Trigonometric functions.

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**UNIT-IV**

Linear transformations, Differentiation, Contraction principle, Inverse function theorem. (9.1 to 9.25 of Chapter 9 of the Text Book)

**Learning Outcomes:** Upon completion of this unit, the student will be able to: Compute derivatives and integrals of real valued and vector-valued functions of several variables. Understand and apply the inverse function theorem.

**UNIT-V**

Implicit function theorem, Determinants, Derivatives of higher order, Differentiation of integrals.

(9.26 to 9.29 and 9.33 to 9.43 of Chapter 9 of the Text Book)

**Learning Outcomes:** Upon completion of this unit, the student will be able to: Understand and apply the implicit function theorem. Compute the derivatives of higher order and differentiation of integrals.

**TEXT BOOK:**

Principles of Mathematical Analysis by Walter Rudin, 3rd Edition.

**REFERENCE BOOK:**

Mathematical Analysis by Tom M. Apostal, Narosa Publishing House, 2nd Edition, 1985.

**Course Outcomes:** After completing this course, the student will be able to: Learn about the uniform behaviour of sequences of plane curves and learn the Weierstrass approximation theorem provides techniques to approximate a continuous function on a compact interval with a polynomial while stones generalization explains method for extension of this concept in the context of algebras. The student shall be able to appreciate the role of fixed point theorem in the inverse function theorem. The student is further introduced to the way in which the inverse function theorem is involved while proving the famous implicit function theorem.

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**CODE: M 2.2(21) (NR)**

M.Sc. DEGREE EXAMINATION, MARCH/APRIL 2021

Second Semester

Mathematics

Paper II – ANALYSIS –II (MODEL PAPER)

Time: Three hours                                                    Maximum : 70 marks

Answer ONE question from each Unit.                                  (5 × 14 = 70 Marks)

**UNIT I**

1. (a) State and prove Cauchy criterion for uniform convergence of sequence of functions.

   (b) Suppose $\{f_n\}$ is a sequence of functions defined on $E$, and suppose $|f_n(x)| \le M_n$ $(x \in E,\ n = 1,2,3,.....)$. Then prove that $\sum f_n$ converges uniformly on $E$ if $\sum M_n$ Converges.

**(OR)**

2 (a) If $\{f_n\}$ is a sequence of continuous functions on $E$, and if $f_n \to f$ uniformly on $E$, then prove that $f$ is continuous on $E$.

   (b) Let $\alpha$ be monotonically increasing on $[a,b]$. Suppose $f_n \in \Re(\alpha)$ on $[a,b]$, for n = 1,2,3,...... and suppose $f_n \to f$ uniformly on $[a,b]$. Then prove that $f \in \Re(\alpha)$ on $[a,b]$, and $\int_a^b f\,d\alpha = \lim_{n\to\infty}\int_a^b f_n\,d\alpha$.

**UNIT II**

3 (a) Prove that there exists a real continuous function on the real line which is nowhere differentiable.

   (b) If $\{f_n\}$ is a pointwise bounded sequence of complex functions on a countable set $E$, then prove that $\{f_n\}$ has a subsequence $\{f_{n_k}\}$ such that $\{f_{n_k}(x)\}$ converges for every $x$ in $E$.

**(OR)**

4 (a) If $K$ is a compact metric space, if $f_n \in \mathscr{C}(K)$ for $n = 1,2,3.......$, and if $\{f_n\}$ converges uniformly on $K$, then prove that $\{f_n\}$ is equicontinuous on $K$.

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(b) If $f$ is a continuous complex function on $[a,b]$, then prove that there exists a sequence of polynomials $P_n$ such that $\lim_{n\to\infty} P_n(x) = f(x)$ uniformly on $[a,b]$.

**UNIT III**

5 (a) Let $\mathscr{B}$ be the uniform closure of an algebra $\mathcal{A}$ of bounded functions. Then prove that $\mathscr{B}$ is a uniformly closed algebra.

(b) State and prove Abel's theorem.

**(OR)**

6 (a) Given a double sequence $\{a_{ij}\}, i = 1,2,3,\ldots\ldots, j = 1,2,3,\ldots\ldots$, suppose that

$\sum_{j=1}^{\infty} |a_{ij}| = b_i \ (i = 1,2,3,\ldots)$ and $\sum b_i$ converges. Then prove that $\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} a_{ij} = \sum_{j=1}^{\infty}\sum_{i=1}^{\infty} a_{ij}$.

(b) If $z$ is a complex number with $|z| = 1$, then prove that there is a unique $t$ in $[0, 2\pi)$ such that $E(it) = z$.

**UNIT IV**

7 (a) Let $r$ be a positive integer. If a vector space $X$ is spanned by a set of $r$ vectors, then prove that $\dim X \le r$.

(b) Suppose $\overline{f}$ maps a convex open set $E \subset \mathbb{R}^n$ into $\mathbb{R}^m$, $\overline{f}$ is differentiable in $E$, and there is a real number $M$ such that $\left\|\overline{f}^1\right\| \le M$ for every $\overline{x} \in E$. Then prove that

$\left|\overline{f}(\overline{b}) - \overline{f}(\overline{a})\right| \le M\left|\overline{b} - \overline{a}\right|$ for all $\overline{a}, \overline{b} \in E$.

**(OR)**

8 State and prove the inverse function theorem.

**UNIT V**

9. State and prove the implicit function theorem.

**(OR)**

10 (a) Prove that a linear operator $A$ on $\mathbb{R}^n$ is invertible if and only if $\det[A] \neq 0$.

(b) Let $E$ be an open set in $\mathbb{R}^2$ and $f$ be defined on $E$. If $f \in \mathscr{C}^{11}(E)$, then prove that $D_{21}f = D_{12}f$.

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**M.Sc. MATHEMATICS, I I SEMESTER**
**M203(21)(NR): MEASURE AND INTEGRATION**
**(With effect from the batch of students admitted during 2021-2022)**

| | | | |
|---|---|---|---|
| Subject Code : | M203(21) | I A Marks | 30 |
| No. of Lecture / Seminar/ Tutorial for week | 06L + 01 S/T | End Exam Marks | 70 |
| | | Total Marks | 100 |

**Course Objectives:** To develop skills and to acquire knowledge on basic concepts of Lebesgue Measure, The Lebesgue Integral, Measurable Functions, $L^p$- spaces, Minkowski inequalities, Holder inequalities, Convergence and completeness.

**UNIT-I**

**Lebesgue Measure:** Introduction, outer measure, Measurable sets and Lebesgue measure, A nonmeasurable sets, Measurable functions, Littlewoods's three principles **(Chapter 3)**

**Learning outcomes**: Upon completion of this unit, the student will be able to: Understand the concept of measure and properties of Lebesgue measure.

**UNIT-II**

**The Lebesgue integral:** The Riemann Integral, The Lebesgue integral of a Bounded function over a set of finite measure, the integral of a non- negative function. The general Lebesgue Integral, Convergence in measure. **(Chapter 4)**

**Learning outcomes**: Upon completion of this unit, the student will be able to: Study the properties of Lebesgue integral and compare it with Riemann integral.

**UNIT-III**

**Differentiation and Integration:** Differentiation of monotone functions, functions of bounded variation, differentiation of an integral, absolute continuity. **(Sections 1 to 4** of **Chapter 5)**

**Learning outcomes**: Upon completion of this unit, the student will be able to: To establish the derivative of the indefinite integral of an integrable function is equal to the integral a.e. To establish the equivalent condition an indefinite integral is absolutely continuous. Jenson inequality becomes a generalization of the inequality between the arithmetic and geometric mean.

**UNIT-IV**

Convex functions, The Classical Banach Spaces: The $L^P$ spaces, The Minkowski and Holder inequalities. (Secton **5** of **chapter 5 & sections 1 to 2 of Chapter 6)**

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**Learning outcomes**: Upon completion of this unit, the student will be able to: To establishes several inequalities involving the $\|.\|_p$ in the $L^p$ spaces.

## UNIT-V

Convergence and completeness, Approximation in $L^P$, Bounded linear functionals on the $L^P$ spaces. **(Sections 3 to 5 of Chapter 6)**

**Learning outcomes**: Upon completion of this unit, the student will be able to: To establishes convergence and completeness, approximation in $L^P$ –space. To find a representation for bounded linear functions.

**TEXT BOOK:** Real Analysis **by H.L. Royden, Third Edition, Pearson Publication.**

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CODE: M203(21)(NR)

# M.Sc. DEGREE EXAMINATION, MARCH 202__
## SECOND SEMESTER
## MATHEMATICS
## Paper III, MEASURE AND INTEGRATION
## Model paper

Time: Three hours  Maximum : 70 marks

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Answer ONE question from each Unit  (5 x 14 = 70 marks)

### UNIT I

1. (a) Let $\{A_n\}$ be a countable collection of sets of real numbers. Then prove that

$m^*(\cup A_n) \le \sum m^*(A_n)$.

(b) State and Prove Egoroff's theorem

(OR)

2. (a) Let $E \subseteq [0,1)$ be a measurable set. Then prove that for each $y \in [0,1)$ the set $E \overset{o}{+} y$ is measurable and $m\left(E \overset{o}{+} y\right) = m(E)$.

(b) Let c be a constant. Let $f$ and g be two measurable real valued defined on the same domain. Then prove that the functions $f$+c, c$f$, $f$+g, g–$f$ and $f$g are also measurable.

### UNIT II

3. (a) Let $f$ be a bounded function defined on [a,b]. If $f$ is Riemann integrable on [a, b] then prove that it is measurable and $R\int_a^b f(x)dx = \int_a^b f(x)dx$.

(b) Let $f$ be a nonnegative function which is integrable over a set E. Then prove that given $\in >0$ there is a $\delta >0$. Such that for energy set $A \subset E$ with $m(A) < \delta$ we have $\int_A f < \in$.

(OR)

4.(a) State and prove Lebesgue convergence theorem

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(b) State and prove monotone convergence theorem

## UNIT III

**5.** State and prove Vitali lemma

**(OR)**

6. Let $f$ be an increasing real-valued function on the interval [a, b]. Then prove that $f$ is differentiable almost everywhere. Also, prove that $f'$ is measurable and

$$\int_a^b f(x)dx \leq f(b) - f(a).$$

## UNIT IV

**7.** (a) If $\phi$ is a continuous function on (a,b) and if one derivative $D^+$ of $\phi$ is non decreasing then prove that $\phi$ is convex.

(b) Prove that $||f+g||_1 \leq ||f||_1 + ||g||_1$

(OR)

8. State and prove Holder inequality.

## UNIT V

9.(a) Prove that $L^p$ is complete ( $1 \leq p < \infty$)

(b) Let $f \in L^p$.Then prove that $\Delta$-approximant $\varphi_\Delta$ of f converges to f in $L^p$ ,i.e., $||f - \varphi_\Delta|| \to 0$, as the length $\Delta$ of the longest sub interval in $\Delta$ approaches zero.

**(OR)**

10. State and prove Riesz Representation Theorem.

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**M.Sc. MATHEMATICS, I I SEMESTER**
**M204(21)(NR): NUMERICAL METHODS**
**(With effect from the batch of students admitted during 2021-2022)**

| Subject Code : | M204(21) | I A Marks | 30 |
|---|---|---|---|
| No. of Lecture / Seminar/ Tutorial for week | 06L + 01 S/T | End Exam Marks | 70 |
| | | Total Marks | 100 |

**Course Objectives:**

The objective of this course includes:

The comprehensive study of the numerical methods for Interpolation of polynomials and Approximation roots of functions and also Integration, The study of numerical methods for solving Linear system of equations and Ordinary differential equations with given initial conditions and ordinary differential equations with given boundary conditions.

**UNIT-I: Interpolation and Approximation:** Introduction, Lagrange and Newton Interpolations, Finite difference Operators, Interpolating polynomials using finite differences, Hermite Interpolations. (Section 4.1 to 4.5 of chapter 4 of [1]).

**Learning outcomes**: Upon completion of this unit, the student will be able to: Apply various Mathematical operations and tasks, such as Interpolation of Polynomials.

**UNIT-II: Numerical Differentiation and Integration:** Introduction, Numerical integration, Methods based on Interpolation, Methods based on Undetermined Coefficients, Composite Integration Methods. (Sections 5.1, 5.6, 5.7, 5.8, 5.9 of chapter 5 of [1] ).
**Learning outcomes**: Upon completion of this unit, the student will be able to: Ability to solve the Problems based on Numerical Integration.

**UNIT-III: Ordinary Differential Equations**: Introduction, Numerical methods, Single step methods, Multi step methods (sections 6.1 to 6.4 of [1]).
**Learning outcomes:** Upon completion of this unit, the student will be able to: find Numerical solution of ordinary differential equations such as Runga-Kutta methods.

**UNIT-IV**: **Ordinary Differential Equations**: Boundary Value Problems: Introduction, Initial Value Problem Method (Shooting Method), Finite Difference Methods. (Sectopms 7.1, 7.2 and 7.3 of Chapter 7 of [1]).
**Learning outcomes**: Upon completion of this unit, the student will be able to find Numerical solution of ordinary differential equations: boundary value problems.

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**UNIT-V**: Numerical Solution of Partial Differential Equations: Introduction, Finite-difference Approximations to Derivatives, Laplace's Equation: Jacobi's Method, Gauss-Seidel Method, Successive Over-Relaxation, Parabolic Equations. (Sections 8.1, to 8.4 of Chapter 8 of [2].

**Learning outcomes**: upon completion of this unit, the student able to find numerical solution of Laplace's equation by using Jacobi's method, Gauss-Seidel Method, Successive Over-Relaxation method.

**TEXT BOOKS:**

[1] "Numerical Methods for Scientific and Engineering Computation", M.K.JAIN,S.R.K. IYANGAR AND R.K. JAIN Third edition, New Age International (p) Limited, New Delhi, 1997.

[2] "Introductory Methods of Numerical Analysis", S. S. Sastry, Published by Prentice Hall of India Pvt.Ltd., Fourth Edition, New Delhi.

**Course outcomes:** Apply numerical methods to obtain approximate solutions to mathematical problems. Derive numerical methods for various mathematical operations and tasks such as interpolation, differentiation, integration, the solution of linear and non linear equations, and the solution of differential equations.

CODE: M 204(21)(NR)

M.Sc. DEGREE EXAMINATION, --- 202---

First Semester

Mathematics

Paper IV- **NUMERICAL METHODS**

MODEL PAPER

Time: Three hours | Maximum: 70 marks

Answer ONE question from each Unit. (5 x 14 = 70 marks)

**UNIT- I**

1(a) The function f(x) = Sin x is defined on the interval [1, 3], then obtain the Lagrange linear interpolating polynomial in this interval and find the bound on the truncation error.

(b) Given that f(0) = 1, f(1) = 3 and f(3) = 55 find the unique polynomial of degree 2 or less that fits the data using the Lagrange's interpolation.

(OR)

2(a) Calculate the $n^{th}$ divided difference of $\frac{1}{x}$, based on the points $x_0, x_1, x_2, \ldots, x_n$.

(b) Find the unique polynomial p(x) of degree 2 or less such that p(1) = 1, p(3) = 27, p(4) = 64. Using Newton divided difference formula.

**UNIT-II**

3(a) Find the approximate value of $I = \int_0^1 \frac{1}{1+x} dx$ by trapezoidal rule and obtain abound for the error's if the exact value of I = log2 = 0.693147 to six decimals.

(b) Evaluate the integral $I = \int_0^1 \frac{1}{1+x} dx$ using Gauss-Legendre three point formula.

(OR)

4(a) Evaluate the integral $I = \int_0^1 (1-x^2)^{3/2} \cos x dx$ using the Gauss-Chebyshev 3-points quadrature rules and Evaluate it also using the Gauss-Legendre 3-point formula.

(b) Evaluate $\int_{-\infty}^{\infty} \frac{e^{-x^2}}{1+x+x^2} dx$ using the Gauss-Hermite two point and three point formulas.

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## UNIT-III

5(a) Solve the initial value problem $u^1 = -2tu^2$, $u(0) = 1$ with h = 0.2 on the interval [0, 2] using the backward Euler method.

(b) Use Euler's mid point method to solve the initial value problem $u^1 = t + u$, $u(0) = 1$ with h = 0.2 and compute u(0.6).

(OR)

6(a) Find the three term Taylor series solution for the third order Blasius equation $w^{111} + ww^{11} = 0$, $w(0) = 0$, $w^1(0) = 0$, $w^{11}(0) = 1$. Find the bound on the error for $t \in [0, 0.2]$.

(b) Solve the system of equations

$u^1 = -3u + 2v,$

$v^1 = 3u - 4v, u(0) = 0, v(0) = 1/2.$

With h = 0.2 in the interval [0, 1]. Use the Euler-Cauchy method.

## UNIT-IV

7. Solve the boundary value problem $y^{11} - 64y + 10 = 0$; $y(0) = y(1) = 0$

By the finite difference method. Compute the value of y(0.5) and compare it with true value.

(OR)

8. Using Shooting method, solve the boundary value problem

$u^{11}(x) = 2uu^1$, , $0 < x < 1$, $u(0) = 0.5$, $u(1) = 1$.

## UNIT-V

9. Solve the Laplace's equation with h = 1/3 over the boundary of a square of unit length with

$u(x, y) = 9x^2y^2$ on the boundary.

(OR)

10. Use the Bender-Schmidt recurrence relation to solve the equation

$\frac{\partial^2 u}{\partial x^2} = 2\frac{\partial u}{\partial x}$; $u(x,0) = 4x - x^2$, $u(0,t) = u(4,t) = 0$.

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CODE: M 205(21) (NR)

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**SYLLABUS**
**(With effect from the batch of students admitted during 2021-2022)**

**II-SEMESTER**

## M 205 - GRAPH THEORY

| Subject Code: | M 205 | I A Marks | 30 |
|---|---|---|---|
| No. of Lecture / Seminar / Tutorial for week | 06L + 01 S/T | End Exam Marks | 70 |
| | | Total Marks | 100 |

**Course Objectives:** The main objective of this course is to introduce the fundamental concepts in graph theory with a sense of some modern applications and it helps the students to solve live problems that can be modeled by graphs.

### UNIT – I

**Paths and Circuits** : Isomorphism, Subgraphs, A puzzle with multi colored cubes, Walks, Paths and circuits, Connected graphs, Disconnected graphs, Components, Euler graphs, Operations on graphs, More on Euler graphs.
(Sections 2.1 to 2.8 of chapter 2 of the Text Book.)

**Learning Outcomes:** Upon completion of this unit, the student will be able to: Understand the basic concepts of graphs and Euler graphs and study the concepts of walks, paths and circuits in a graph.

### UNIT – II

**Hamiltonian Graphs:** Hamiltonian paths and circuits, Traveling salesman problem.
**Trees:** Trees, Some properties of trees, Pendant vertices in a tree, Distance and centers in a tree, Rooted and binary trees, On counting trees.
(Sections 2.9 to 2.10 of Chapter 2 and 3.1 to 3.6 of Chapter 3 of the Text Book.)

**Learning Outcomes:** Upon completion of this unit, the student will be able to: Understand the concepts of Hamiltonian graphs and obtain a solution for Travelling salesman problems. Study the properties of trees, pendent vertices, centers in a tree and also study rooted and binary trees.

### UNIT – III

**Fundamental Circuits:** Spanning trees, Fundamental circuits, Finding all spanning trees of a graph, Spanning trees in weighted Graphs.
**Cut-sets:** Cut-sets, All cut-sets in a graph, Fundamental circuits and cut-sets.
(Sections 3.7 to 3.10 of Chapter 3 and 4.1 to 4.4 of Chapter 4 of the Text Book .)

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**Learning Outcomes:** Upon completion of this unit, the student will be able to: Find a minimal spanning tree for a given weighted graph. Understand the purpose of introduction of concepts like fundamental circuits and cut-sets in a graph.

**UNIT – IV**

**Cut-vertices :** Connectivity and separability, Network flows, 1- Isomorphism, 2-Isomorphism.
**Planar Graphs:** Combinatorial Vs. geometric graphs, Planar graphs, Kuratowski's two graphs, Different representations of a planar graph.
(Sections 4.5 to 4.8 of Chapter 4 and 5.1 to 5.4 of Chapter 5 of the Text Book.)

**Learning Outcomes:** Upon completion of this unit, the student will be able to: Understand the purpose of introduction of concepts like connectivity and separability. Study about Combinatorial Vs. geometric graphs, Planar graphs and also study about the Kuratowski's two graphs.

**UNIT – V**

**Dual Graphs:** Detection of planarity, Geometric dual.
**Vector Spaces of a Graph:** Sets with one operation, Sets with two operations, Modular Arithmetic and Galois fields, Vectors and Vector Spaces, Vector Space associated with a graph, Basis vectors of a graph.
(Sections 5.5 to 5.6 of Chapter 5 and 6.1 to 6.6 of Chapter 6 of the Text Book.)

**Learning Outcomes:** Upon completion of this unit, the student will be able to: Understand the detection of planarity and geometrical dual. Study modular arithmetic and Galois fields and also study the vector space associated with a graph.

**TEXT BOOK:**

" Graph Theory with Applications to Engineering and Computer Science" by 'NARSINGH DEO', Prentice Hall of India, Pvt Ltd., New Delhi, 1993.

**Course Outcomes:** After completing this course, the student will be able to: Understand the basic concepts of graphs, directed graphs, weighted graphs, trees, minimal spanning trees for a given graphs, Eulerian graphs, Hamiltonian graphs and apply the shortest path algorithm to solve some real life problems.

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(M 2.5(21)(NR))

M.Sc. DEGREE EXAMINATION, OCTOBER/NOVEMBER 2021

Second Semester

Mathematics

Paper V – GRAPH THEORY (MODEL PAPER)

Time: Three hours

Maximum : 70 Marks

Answer ONE question from each Unit.

(5 x 14 = 70 Marks)

UNIT I

1. (a) Show that a graph $G$ is disconnected if and only if its vertex set $V$ can be partitioned into two nonempty disjoint subsets $V_1$ and $V_2$ of $V$ such that there exists no edge in $G$ whose one end vertex is in subset $V_1$ and the other end vertex is in subset $V_2$.

   (b) Show that a simple graph with $n$ vertices and $k$ components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.

(OR)

2 (a) Show that in a connected graph $G$ with exactly $2k$ odd vertices, there exist $k$ edge – disjoint subgraphs such that they together contain all edges of $G$ and that each is a unicursal graph.

   (b) Show that a connected graph $G$ is an Euler graph if and only if it can be decomposed into circuits.

UNIT II

3 (a) Show that in a complete graph with $n$ vertices there are $\frac{n-1}{2}$ edge – disjoint Hamiltonian circuits, if $n$ is an odd number $\geq 3$.

   (b) Show that a graph $G$ is a tree if and only if there is one and only one path between any two vertices in $G$.

(OR)

4 (a) Show that a tree with $n$ vertices has $n-1$ edges.

   (b) Show that every tree has either one or two centers.

UNIT III

5 (a) Show that every connected graph has atleast one spanning tree.

   (b) Show that a connected graph $G$ is a tree if and only if adding an edge between any two vertices in $G$ creates exactly one circuit.

(OR)

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6 (a) Define a cut – set in a graph. Show that in a connected graph $G$, any minimal set of edges containing atleast one branch of every spanning tree of $G$ is a cut – set.

(b) Show that the ring sum of any two cut – sets in a graph is either a third cut – set or an edge – disjoint union of cut – sets.

**UNIT IV**

7 (a) Show that the edge connectivity of a graph $G$ cannot exceed the degree of the vertex with the smallest degree in $G$.

(b) Show that the maximum flow possible between two vertices $a$ and $b$ in a network is equal to the minimum of the capacities of all cut – sets with respect to $a$ and $b$.

**(OR)**

8 (a) Show that the complete graph of five vertices in nonplanar.

(b) Show that a graph can be embedded in the surface of a sphere if and only if it can be embedded in a plane.

**UNIT V**

9 (a) Prove that a necessary and sufficient condition for a graph $G$ to be planar is that $G$ does not contain either of Kuratowski's two graphs or any graph homeomorphic to either of them.

(b) Prove that a graph has a dual if and only if it is planar.

**(OR)**

10 (a) Show that the ring sum of two circuits in a graph $G$ is either a circuit or an edge – disjoint union of circuits.

(b) Show that the set consisting of all the cut-sets and the edge – disjoint unions of cut – sets including the empty set in a graph $G$ is an abelian group under the ring sum operation.

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## M.Sc. MATHEMATICS III SEMESTER
## M301: RINGS AND MODULES

| Subject Code : | M301 | I A Marks | 30 |
|---|---|---|---|
| No. of Lecture / Seminar/ Tutorial for week | 06L + 01 S/T | End Exam Marks | 70 |
| | | Total Marks | 100 |

**Course Objectives:** To develop skills and to acquire knowledge on some advanced concepts of Modern Algebra i.e. different algebraic structures, Modules, Prime ideals, prime radical, Jacobson radical in commutative rings, complete ring of quotients, Prime ideal spaces.

**UNIT-I:** Rings and related Algebraic systems, Subrings, Homomorphisms, Ideals. (Sections 1.1, 1.2 of chapter 1).

**Learning outcomes**: Upon completion of this unit, the student will be able to: Understand the concepts of commutative ring theory and special structures like Boolean algebras and Boolean rings. Know the relations between ring, Boolean algebra and lattice.

**UNIT-II** : Modules, Direct products and Direct sums, Classical Isomorphism Theorems. (Sections 1.3, 1.4 of chapter 1).

**Learning outcomes**: Upon completion of this unit, the student will be able to study: Classical isomorphism theorems and some properties of direct sum, product of rings and modules.

**UNIT-III:** Prime ideals in Commutative Rings, Prime ideals in Special Commutative Rings. (Sections 2.1, 2.2 of Chapter 2).

**Learning outcomes**: Upon completion of this unit, the student will be able to: Understand the concept of Prime ideals, maximal ideals of commutative rings, Prime radical and Jacobson radical.

**UNIT-IV:** The Complete Ring of Quotients of a Commutative Ring (Section 2.3 of Chapter 2).

**Learning outcomes**: Upon completion of this unit, the student will be able to: Two methods Applied to any integral domain to Construct its field of quotients, one method is applied to any commutative ring to construct its classical ring of quotients.

**UNIT-V:** Ring of quotients of Commutative Semi Prime Rings, prime ideal spaces. (Sections 2.4& 2.5 of Chapter 2).

**Learning outcomes**: Upon completion of this unit, the student will be able to: Study the Wedderburn – Artin theorem and its applications and Prime ideal spaces.

**TEXT BOOK:** "Lectures on Rings and Modules", J. Lambek, Blaisdell Publications.

**Course outcome**: The student attains more mathematical sophistication extending the concepts of rings introduced in the introductory course 101.

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Course Code: (M 3.1)(12)

# ACHARYA NAGARJUNA UNIVERSITY
## M.Sc. DEGREE EXAMINATION
### Third Semester, Mathematics
### Paper I – RINGS AND MODULUS
### MODEL PAPER

Time: Three hours | Max. Marks: 70

Answer ONE Question from each Unit. 5 x 14 = 70 marks

## UNIT I

1. (a) If θ is a reflexive homomorphic relation on a ring then prove that θ is a congruence relation.
   (b) Prove that there is a one to one correspondence between the ideals K and the congruence relations θ of a ring R such that $r - r' \in K \Leftrightarrow r\,\theta\,r'$.

Or

2. (a) Define closure operation on a complete lattice. Prove that the set of closed elements from a complete lattice on a complete lattice with a closure operation.
   (b) If ϕ is a homomorphism of a ring R into another ring then prove that $\phi(R) \cong {}^{R}/_{\phi^{-1}(0)}$.

## UNIT II

3. (a) State and prove Schreire's theorem for chains of sub modules of an a module $A_R$.
   (b) If B and C are sub modules of module A then prove that $B + {}^{C}/_{B} \cong {}^{C}/_{B \cap C}$.

Or

4. (a) Let B be a sub modules of $A_R$. Then prove that A is Artinian if and only if B and ${}^{A}/_{B}$ are Artinian.
   (b) Prove that control idempotents of a ring R from a Boolean algebra B (R).

## UNIT III

5 (a) Prove that the radial R a ring consists of all elements r of the ring such that 1 – rx is a unit for all x in the ring.
  (b) Prove that the ring ${}^{R}/_{\text{rad } R}$ is semi prime.

Or

6 (a) Prove that every commutative regular ring is semi primitive.
  (b) Let R be a commutative ring. Then prove that the following conditions are Equivalent:
    (i) Every non zero divisor is nilpotent
    (ii) R has a minimal prime ideal P and this contains all zero divisors.

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## UNIT IV

7 Let R be a sub ring of the commutative ring S. Then prove the following statements are Equivalent:

(a) S is a ring of quotients of R.

(b) For all $0 \neq s \in S$, $s^{-1}R$ is a dense ideal of R and $s(s^{-1}R) \neq (0)$.

(c) There exists a monomorphism of S into $\phi(R)$ which induces the canonical monomorphism of R into $\phi(R)$.

Or

8 (a) State and prove the properties of dense ideals

(b) If R is a commutative ring then prove that the system $(F,0,1,-1,+,.) / \theta = Q(R)$ is also a commutative ring.

## UNIT IV

9. In any commutative ring R prove that

(i) $K \subset J \Rightarrow J^* \subseteq \kappa^*$

(ii) $K \subseteq \kappa^{**}$

(iii) $\kappa^{***} = \kappa^*$ where $\kappa^*$ denotes the annihilator of $\kappa$.

Or

10 (a) If R is a Boolean ring then Q(R) is a Boolean ring

(b) If R is an atomic Boolean algebra then prove that its completion is isomorphic to the algebra of all sets of atoms of R.

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**M 302 (NR)**

**DEPARTMENT OF MATHEMATICS**
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**M.Sc., Mathematics**
**SYLLABUS**
**(With effect from the batch of students admitted during 2021-2022)**
**III-SEMESTER**
**M 302 – COMPLEX ANALYSIS**

| Subject Code: | M 302 | I A Marks | 30 |
|---|---|---|---|
| No. of Lecture / Seminar / Tutorial for week | 06L + 01 S/T | End Exam Marks | 70 |
| | | Total Marks | 100 |

**Course Objectives:**To introduce and develop the fundamental concepts of complex analysis such as analytic functions, Cauchy-Riemann equations and harmonic functions etc., To study Cauchy integral formula, general form of Cauchy theorem, the Fundamental theorem of Algebra and Maximum modulus Principle and acquire skills of contour integration to evaluate definite integrals involving sines and cosines via residue calculus.

**UNIT-I**

Sums and products, basic algebraic properties, further properties, vectors and moduli, complex conjugates, exponential form, products and powers in exponential form, arguments of products and quotients, Roots of complex numbers- examples – Regions in the complex plane.

**(Sections 1 to 11 of Text Book)** **(Questions not to be given in Sections 1 to 11)**

Functions of a complex variable, mappings, mappings by the exponential function, limits, Theorems on limits, limits involving the point at infinity, continuity, derivatives, Differentiation formulas, Cauchy-Riemann equations, sufficient conditions for differentiability, polar co-ordinates, Analytic functions. **(Sections 12 to 25 of Text Book)**

**Learning outcomes:** Upon completion of this unit, the student will be able to represent complex numbers algebraically and geometrically and understand Analytic functions, Cauchy-Riemann equations and verify Complex functions for analycity.

**UNIT-II**

Harmonic functions, Uniquely determined Analytical functions, Reflection principle.

The exponential function, the logarithmic functions, branches and derivatives of logarithms, contours, contour integrals, Some examples - Examples with branch cuts, upper bounds for moduli of contour integrals, anti-derivatives, Proof of the Theorem (45), Cauchy-Goursat theorem, Proof of the Theorem (47), simply connected domains, multiply connected domains.

**(Sections 26 to 31 & 39 to 49 of Text Book)**

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**Learning outcomes:** Upon completion of this unit, the student will be able to understand Harmonic functions and Cauchy-Goursat theorem to simply connected domains and Cauchy-Goursat theorem to multiply connected domains.

## UNIT-III

Cauchy integral formula, An extension of the Cauchy integral formula – Some consequences of the extension. Liouvelli's theorem and the fundamental theorem of Algebra, maximum modulus principle. Convergence of sequences, Convergence of series, Taylor series.

**(Sections 50 to 59 of Text Book)**

**Learning outcomes:** Upon completion of this unit, the student will be able to evaluate Complex integrals by applying Cauchy integral formula and to understand Liouvelli's theorem and the fundamental theorem of Algebra and maximum modulus principle.

## UNIT-IV

Laurent series, absolute and uniform convergence of power series, continuity of sums of power series, integration and differentiation of power series, uniqueness of series representations, Isolated singular points, Residues, Cauchy residue theorem, Residue at infinity – The three types of isolated singular points. **(Sections 60 to 72 of Text Book)**

**Learning outcomes:** Upon completion of this unit, the student will be able to represent functions as Laurent series and to understand residues and Cauchy residue theorem.

## UNIT-V

Residues at poles, Examples, zeros of analytic functions, zeros and poles, behavior of a function near isolated singular points. Evaluation of improper integrals, Examples - Improper integrals from Fourier analysis, Jordan's Lemma, definite integrals involving Sines and Cosines, Argument Principle, Rouche's Theorem. **(Sections 73 to 81 & 85 to 87 of Text Book)**

**Learning outcomes:** Upon completion of this unit, the student will be able to compute integrals using residues and to understand Argument principle and Rouche's theorem.

**Text Book:** Complex variables and Applications, James Ward Brown, Ruel V.Churchill, Mc Graw Hill, Eighth Edition, 2009.

**Reference Books:**

Complex Variables, H.Silvermen

Complex Variables by H.S.Kasana, Prentice Hall of India

Complex Variables by Murrey Rspiegel, Scheam's Outline series.

**Course outcomes:** Upon completion of this course, the student will be able to define and analyze limits and continuity for functions of complex variables, Cauchy-Riemann equations, analytic functions and entire functions. Evaluate complex contour integrals, the Cauchy integral formula and represent functions as Taylor and Laurent series, classify singularities and poles, find residues and evaluate complex integrals using the residue theorem.

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M.Sc. DEGREE EXAMINATION

Third Semester

Mathematics

Paper II – COMPLEX ANALYSIS (MODEL PAPER)

Time: Three hours | Maximum : 70 marks.

Answer ONE question from each Unit.

**UNIT I**

1. (a) Suppose that $f(z)=u(x,y)+i\,v(x,y)$, $z_0=x_0+i\,y_0$ and $w_0=u_0+i\,v_0$. Then prove that $\lim_{z\to z_0} f(z)=w_0$ if and only if $\lim_{(x,y)\to(x_0,y_0)} u(x,y)=u_0$ and $\lim_{(x,y)\to(x_0,y_0)} v(x,y)=v_0$.

   (b) Examine the differentiability of the function $f(z)=|z|^2$.

**(OR)**

2 (a) Let the function $f(z)=u(x,y)+i\,v(x,y)$ be defined throughout some $\in$ – neighbourhood of a point $z_0=x_0+i\,y_0$. Suppose that the first order partial derivatives of the functions $u$ and $v$ with respect to $x$ and $y$ exist everywhere in that neighbourhood and that they are continuous at $(x_0,y_0)$. If the partial derivatives satisfy the Cauchy-Riemann equations $u_x=v_y$, $u_y=-v_x$ at $(x_0,y_0)$ then prove that the derivative $f^1(z_0)$ exists.

   (b) Suppose that in a domain D, a function $f(z)=u(x,y)+i\,v(x,y)$ is analytic and its modulus $|f(z)|$ is constant. Show that $f(z)$ must also be constant in D.

**UNIT II**

3 (a) Prove that if a function $f(z)=u(x,y)+i\,v(x,y)$ is analytic in a domain D, then its component functions $u$ and $v$ are harmonic in D.

   (b) Show that $u(x,y)=\dfrac{y}{x^2+y^2}$ is harmonic in some domain and find a harmonic conjugate $v(x,y)$.

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**(OR)**

4 (a) State and prove Cauchy-Goursat theorem.

(b) By finding an anti-derivative, evaluate $\int_C \frac{dz}{z^2}$, where '$C$' is the positively oriented circle $z = 2e^{i\theta}, -\pi \le \theta \le \pi$.

**UNIT III**

5 (a) State and prove Cauchy integral formula.

(b) Prove that if a function $f$ is analytic at a point, then its derivatives of all orders are also analytic functions at that point.

**(OR)**

6 (a) State and prove Liouville's theorem.

(b) Show that when $z \ne 0$, $\frac{\sin(z^2)}{z^4} = \frac{1}{z^2} - \frac{z^2}{3!} + \frac{z^6}{5!} - \frac{z^{10}}{7!} + .....$

**UNIT IV**

7 (a) Find the Laurent series expansion of $f(z) = \frac{-1}{(z-1)(z-2)}$ throughout the annulus $1 < |z| < 2$.

(b) Prove that, if a power series $\sum_{n=0}^{\infty} a_n z^n$ converges when $z = z_1 (z_1 \ne 0)$, it is absolutely convergent for every value of $z$ such that $|z| < |z_1|$.

**(OR)**

8 (a) State and prove Cauchy residue theorem.

(b) Determine the order $m$ of each pole and find the corresponding residue $B$ of $f(z) = \frac{\exp(2z)}{(z-1)^2}$.

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## UNIT V

9. (a) Determine the order $m$ of each pole and find the corresponding residue $B$ of

$$f(z)=\frac{z^3+2z}{(z-i)^3}.$$

(b) Find the value of the integral $\int_{-\infty}^{\infty}\frac{x\sin x}{x^2+2x+2}dx.$

**(OR)**

10 (a) State and prove Rouche's Theorem.

(b) Show that $\int_{0}^{2\pi}\frac{d\theta}{1+a\sin\theta}=\frac{2\pi}{\sqrt{1-\omega^2}},\ -1<\omega<1.$

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**M.Sc. MATHEMATICS-III SEMESTER**
**M303(21)(NR): FUNCTIONAL ANALYSIS**
**(With effect from the batch of students admitted during 2021-2022)**

| | | | |
|---|---|---|---|
| Subject Code : | M303(21) | I A Marks | 30 |
| No. of Lecture / Seminar/ Tutorial for week | 06L + 01 S/T | End Exam Marks | 70 |
| | | Total Marks | 100 |

**Course Objectives:** To introduce basic concepts of Functional Analysis namely normed spaces, bounded linear functionals, and study their applications and also to introduce fundamental results in Functional Analysis namely Hahn-Banach Theorem, open mapping theorem and closed graph theorem and study their applications.

**UNIT-I**

Review of properties of Metric spaces (Chapter-1);Vector space - Normed spaces, Banach space - Further properties of normed spaces - Finite dimensional normed spaces-compactness and finite Dimension.
(2.1 to 2.5 of Chapter 2)

**Learning outcomes**: Upon completion of this unit, the student will be able to: Understand basic properties of finite dimensional normed spaces.

**UNIT-II**

Linear operators – Bounded and continuous linear operators – Linear functionals – Linear operators and functionals on Finite dimensional spaces – Normed spaces of operators, Dual Space.(2.6 to 2.10 of Chapter 2)

**Learning outcomes**: Upon completion of this unit, the student will be able to: Analise bounded linear operators and functionals of finite dimensional normed spaces.

**UNIT-III**

Banach fixed point theorem – Applications of Banach fixed point theorem to linear equations and differential equations–Zorn's lemma - Hann Banach theorem – Hann Banach theorem to complex vector spaces and normed spaces.
(5.1 to 5.3 of Chapter5 and 4.1 to 4.3 of Chapter 4)

**Learning outcomes**: Upon completion of this unit, the student will be able to: Demonstrate the knowledge of Banach fixed point theoremand apply it to linear and differential equations.Also analyze the Hahn-Banach theorem is different spaces.

**UNIT- IV**

Applications to bounded linear functionals onC[a, b] - Adjoint Operator – Reflexive spaces –Category theorem and Uniform boundedness theorem.
(Sections 4.4 to 4.7 of Chapter 4)

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**Learning outcomes**: Upon completion of this unit, the student will be able to: Describe reflexive spaces, Category theoremand uniform boundedness principle,

**UNIT- V**

Strong and weak convergence - Convergence of sequences of operators and functionals – Open mapping theorem – Closed graph theorem
(Sections 4.8,4.9,4.12 and 4.13 of Chapter 4).

**Learning outcomes**: Upon completion of this unit, the student will be able to: Describe strong and weak convergences, open mapping theorem and closed graph theorem.

**TEXT BOOK:**
Introductory Functional analysis with applications by Erwin Kreyszig, John Willy and sons.

**Reference Books:**

1. Introduction to Topology and Modern Analysis by G.F. Simmons, McGraw Hill Book Company, New York International student edition.
2. Introduction to Functional Analysis, by A. E. Taylor, Wiley, New York, 1958.

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CODE: M303(21)(NR)

# M.Sc. DEGREE EXAMINATION,
## THIRED SEMESTER
## MATHEMATICS
## Paper –III, FUNCTIONAL ANALYSIS
## Model paper

Time: Three hours Maximum : 70 marks

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Answer ONE question from each Unit (5 x 14 = 70 marks)

### UNIT I

1. (a) Prove that every finite dimensional subspace Y of a normed space X is complete.

   (b) Prove that in a finite dimensional normed space X, any subset M of X is compact if and only if M is closed and bounded.

Or

2. (a) If a normed space X has the property that its closed unit ball is compact then prove that X is finite dimensional.

   (b) State and prove Riesz's lemma.

### UNIT II

3. (a) Let T :$\mathcal{D}$ (T) $\rightarrow$ Y be a linear operator, where $\mathcal{D}$ (T) is a subspace of a normed space X and Y is also a normed space. Then prove that T is continuous if and only if T is bounded

   (b)If a normed space X is finite dimensional then prove that every linear operator on X is bounded.

Or

4. (a) Let T :$\mathcal{D}$ (T) $\rightarrow$ Y be a bounded linear operator, $\mathcal{D}$ (T) a subspace of a normed space X and Y a Banach space. Then prove that T can be extended to a bounded linear operator S defined on the closure of$\mathcal{D}$ (T) such that the norm of S is equal to the norm of T.

   (b) If X and Y are normed spaces over K and Y is a Banach space then prove that B(X, Y) is also a Banach space.

### UNIT III

5. (a) State and prove Banach fixed point theorem.

   (b) State and prove Hahn-Banach theorem for complex vector spaces.

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Or

6. (a) Prove that every vector space $X \neq \{0\}$ has a Hamel basis.

(b) Let X be a normed space and let $x_0 \neq 0$ be any element of X. Then Prove thatthere exists a bounded linear functional $\tilde{f}$ on X such that $\|\tilde{f}\| = 1, \tilde{f}(x_0) = \|x_0\|$.

UNIT IV

7. State and prove Riesz's theorem for functionals on C[a, b].

Or

8. (a) Prove that if the dual space $X^1$ of a normed space X is separable, then prove that X itself is separable.

(b) State and prove uniform boundedness theorem.

UNIT V

9. (a) Let $(x_n)$ be a sequence in a normed space X. If dim $X < \infty$ then prove that weak convergence of $(x_n)$ in X implies strong convergence of $(x_n)$ in X.

(b) Prove that a sequence $(T_n)$ of operators inB(X, Y), where X and Y are Banach spaces, is strongly operator convergent if and only if

(i) the sequence $(\|T_n\|)$ is bounded;

(ii) the sequence $(T_n(x))$ is Cauchy in Y for every x in a total subset M of X.

Or

10. State and prove open mapping theorem.

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**M.Sc. MATHEMATICS-III SEMESTER**
**M304(A)(21): FUZZY SETS AND THEIR APPLICATIONS**
**(With effect from the batch of students admitted during 2021-2022)**

| Subject Code : | M304(A)(21) | I A Marks | 30 |
|---|---|---|---|
| No. of Lecture / Seminar/Tutorial for week | 06L + 01 S/T | End Exam Marks | 70 |
| | | Total Marks | 100 |

**Course Objectives:** The objective of this course is to teach the students the need of fuzzy sets ,operations on fuzzy sets, arithmetic operations on fuzzy sets and fuzzy relations.

**UNIT-1:** From Classical (Crisp) sets to Fuzzy sets: **Introduction, Crisp Sets:** An overview, Fuzzyset: Basic types, Fuzzy sets: Basic Concepts, Characteristics and significance of the paradigm shift. (Sections 1.1-1.5 of Chapter -1 of text book)

**Fuzzy sets versus Crisp sets**: Additional Properties of α-cuts, Representations of Fuzzy sets,Extension principle for Fuzzy sets (Sections 2.1-2.3 of Chapters 2 of Text book).

**Learning outcomes**: Upon completion of this unit, the student will be able to: Understand the basicconcepts of fuzzy sets, properties of α-cut sets and extension principle of fuzzy sets.

**UNIT – II: Operations on Fuzzy sets:** Types of Operations, Fuzzy Compliments, Fuzzy Intersections: t-Norms, Fuzzy unions: t-Conorms, Combinations of operations, AgreegationOperations (Sections 3,1-3.6 of Chapter-3 of Text book).

**Learning outcomes**: Upon completion of this unit, the student will be able to: Describe fuzzycompliments, fuzzy intersections and fuzzy unions.

**UNIT- III**: Fuzzy Arithmetic: Fuzzy Numbers, Linguistic Variables, Arithmetic Operations on Intervals, Arithmetic Operations on Fuzzy numbers, Lattice of fuzzy numbers, Fuzzy equations(Sections 4.1-4.6 of Chapter 4 of Text book).

**Learning outcomes**: Upon completion of this unit, the student will be able to: Understand theconcept of fuzzy arithmetic.

**UNIT-IV: Fuzzy Relations:** Crisp versus fuzzy relations, Projections and Cylindric Extensions,Binary Fuzzy Relations, Binary Relations on a Single set, Fuzzy Equivalence Relations, Fuzzy Compatibility Relations. (Sections 5.1-5.6 of Chapter 5 of Text book).

**Learning outcomes**: Upon completion of this unit, the student will be able to: Determine the difference between crisp relations, fuzzy relations and understand the concepts of fuzzy compatibilityrelations and fuzzy ordering relations.

**UNIT-V:**

Fuzzy Ordering Relations, Fuzzy Morphisms, Sup – i Compositions of Fuzzy Relations, Inf- $\omega_i$ Compositions of fuzzy Relations.(Sections 5.7-5.10 of Chapter 5 of Text book).

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**Learning outcomes**: Upon completion of this unit, the student will be able to: Fuzzy Ordering Relations, Fuzzy Morphisms, Sup – i Compositions of Fuzzy Relations, Inf- $\omega_i$ Compositions of fuzzy Relations.

**PRESCRIBED BOOK: "Fuzzy sets and Fuzzy Logic, Theory and Applications"**,G.J.Klir & B.YUAN, Prentice - Hall of India Pvt. Ltd., New Delhi., 2001.

**Course outcomes:** After completing this course, the student shall be able to: Understand the basic concepts of fuzzy sets, fuzzy arithmetic and fuzzy relations. Construct the appropriate fuzzy numberscorresponding to uncertain and imprecise collected data and also determine the concepts of fuzzy compatibility relations, fuzzy ordering relations and fuzzy morphisms.

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CODE: M304(A)(21)

# M.Sc. DEGREE EXAMINATION, MARCH 202__

## THIRD SEMESTER

## MATHEMATICS

## Paper –IV, FUZZY SETS AND THEIR APPLICATIONS

Model paper

Time: Three hours | Maximum : 70 marks

Answer ONE question from each Unit | (5 x 14 = 70 marks)

### UNIT-I

1 a) A fuzzy set A on R is convex iff $A(\lambda x_1 + (1-\lambda)x_2) \geq \min[A(x_1), A(x_2)]$ for all $x_1, x_2 \in \mathbf{R}$ and all $\lambda \in [0, 1]$, where min denotes the minimum operator.

b) Let A, B be two fuzzy sets of a universal set X. Prove that: $(A\Delta B)\Delta C = A\Delta(B\Delta C)$;

(OR)

2 a) State and prove First Decomposition Theorem

b) Let $f : X \rightarrow Y$ be an arbitrary crisp function. Then, for any $A_i \in \mathbf{F}(X)$ and any $B_i \in \mathbf{F}(Y), i \in I$, the following properties of functions obtained by the extension principle hold:

(i) $f(A) = \emptyset$ iff $A = \emptyset$

(ii) $f(\cup_{i \in I} A_i) = \cup_{i \in I} f(A_i)$;

### UNIT-II

3 a) State and prove second characterization theorem of fuzzy compliments.

b) For all a, b $\in [0, 1]$, then prove that $i_{min}(a, b) \leq i(a, b) \leq \min(a, b)$, where $i_{min}$ denotes the drastic intersection.

(OR)

4 a) Given a t-norm i and an involutive fuzzy complement c, the binary operation u on [0, 1] defined by $u(a, b) = c(i(c(a), c(b)))$ for all a, b $\in [0, 1]$ is a t-conorm such that (i, u, c) is a dual triple.

b) Let (i, u, c) be a dual triple that satisfies the law of excluded middle and the law of contradiction. Then, (i, u, c) does not satisfy the distributive laws.

### UNIT-III

5 a) Let $* \in \{+, -, ., /\}$, and let A, B denote continuous fuzzy numbers. Then, the fuzzy set A * B defined by $(A * B)(z) = \sup_{z = x * y} \min[A(x), B(y)]$ is a continuous fuzzy number.for all $z \in R$.

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(OR)

6 a) Let MIN and MAX, be binary operations on R. Then Prove that for any A, B, C $\in$R MIN [A, MAX (A, B)] = A.

b) Solve the equation A+ X = B.

## UNIT-IV

7 a) Prove that the Standard(or max –min) composition is associative and its inverse is equal to the reverse composition of the inverse relation.

b) Prove tha tproperties of symmetry,reflexivity and transitivity are preserved under inversion for both crisp and fuzzy relations.

(OR)

8 a) Write transitive closure algorithm. By using the algorithm, Determine the transitive max-min closure $R_T(X,X)$ for a Fuzzy relation R(x,x) defined by the membership matrix

$$R=\begin{bmatrix} .7 & .5 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & .4 & 0 & 0 \\ 0 & 0 & .8 & 0 \end{bmatrix}$$

## UNIT-V

9 a) For any fuzzy relation R on $X^2$, the fuzzy relation $R_{T(i)} = \bigcup_{n=1}^{\infty} R^{(n)}$ is the i-transitive closure of R.

b) Let R be a reflexive fuzzy relation on $X^2$, where $|X| = n \geq 2$. Then prove that $R_{T(i)} = R^{(n-1)}$.

(OR)

10 a) Prove that i (a, b) $\leq$ d iff $w_i$ (a, d) $\geq$ b; where i is a continuous t-norm i.

b) Let P(X,Y),$Q_1$,(Y,Z),$Q_2$,(Y,Z)and R(Z,V) be fuzzy relations if $Q_1 \subseteq Q_2$ then show that Po $Q_1 \subseteq$ Po $Q_2$ and $Q_1$o R $\subseteq$ $Q_2$o R.

**M.Sc. MATHEMATICS-III SEMESTER**
**M304(B)(21)(NR): SEMI GROUPS**
**(With effect from the batch of students admitted during 2021-2022)**

| | | | |
|---|---|---|---|
| Subject Code : | M304(B)(21) | I A Marks | 30 |
| No. of Lecture / Seminar/ Tutorial for week | 06L + 01 S/T | End Exam Marks | 70 |
| | | Total Marks | 100 |

**Course Objectives:** To introduce the Concepts of Semigroups, Homogenic Semigroups, Free Semigroups, Ideals, Regular Semigroups, Simple and Q-Simple Semigroups, and their relatedtheories to develop working knowledge on these concepts.

**UNIT – I**
Basic Definitions, Monogenic Semigroups, Ordered Sets, Semilattices and Lattices, BinaryRelations, Equivalences. ( Sections 1.1-1.4 of Chapter- I).

**Learning outcomes**: Upon completion of this unit, the student will be able to: Understand basicdefinitions of Semigroups, Semilattices and Lattices, and their basic Results.

**UNIT – II**
Congruences , Free Semigroups and Monoids:Presentations, Ideals and Rees Congruences, Lattices of Equivalences and Congruences equivalences. ( Sections 1.5 to1.8 of Ch. 1).

**Learning outcomes**: Upon completion of this unit, the student will be able to: Understand FreeSemigroups, Ideals and Lattice's of equivalences.

**UNIT - III**
Green's equivalences ,The structure of D. Classes – Regular D-Classes, Regular Semigroups.(Sections 2.1-2.4 of Chapter 2 ).

**Learning outcomes**: Upon completion of this unit, the student will be able to: Green's equivalences, find Structure of D.Classes and regular semi groups.

**UNIT –IV**
Simple and Q – Simple Semigroups; Principle Factors, Rees's Theorem, Completely Simple semigroups, Isomorphism and Normalization. (Sections 3.1-3.4 of Chapter 3).

**Learning outcomes**: Upon completion of this unit, the student will be able to: Analyze Simple and0-Simple Semigroups, and Rees's Theorem and Isomorphism.

**UNIT –V**
Congruences on Completely 0 – Simple semigroups, The Lattice of Congruences on a Completely 0 –Simple Semigroup, Finite Congruence- Free Semigroups. (Sections 3.5-3.7 of Chapter 3).

**Learning outcomes**: Upon completion of this unit, the student will be able to: Describe Congruences on Completely O-Simple Semigroups and Finite Congruences.

**TEXT BOOK: "An Introduction to Semigroup Theory"**, J.M. Howie, Academic Press.

**Course outcomes:** The student realizes the richness of properties enjoied by Semigroups, an algebraicstructure with fewer facilities than Groups.

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CODE: M304(B)(21)

M.Sc. DEGREE EXAMINATION, MARCH 202__

THIRED SEMESTER

MATHEMATICS

Paper –IV, SEMI GROUPS

Model paper

Time: Three hours | Maximum : 70 marks

Answer ONE question from each Unit (5 x 14 = 70 marks)

UNIT-I

1. a) In a periodic semi group every element has a power which is idempotent . hence in a periodic semi group there is atleast one idempotent.

   b) Let Y be a non empty subset of a partially ordered set X .Then

   i) Y has at one minimum element.

   ii) if Y is toatallly ordered , then the terms " minimal " and "minimum " are equivalent.

(OR)

2 a ) prove that ( B(X),o) is a semi group.

   b) If Ø:X→Y is a mapping then prove that Øo $Ø^{-1}$ is an equivalence .

UNIT-II

3 a) A relation ρ on a semi group S is a congruence if and only if it is both a left and a right congruence.

   b) Let R be a left and right compatible relation on a semigroup S then prove that $R^n$(=RoRo…oR) is left and right compatible for every n≥1.

(OR)

4 a) Prove that a modular lattice is semimodular.

   b) Prove that the lattice (Ɛ(X)≤, ∩,U) of equivalence es on asset X is semi modular.

UNIT-III

5 a) Prove that the relation L and R commute .

   b) If a,b are **D** –equivalent elementsin a semi group S then show that $|H_a|=|H_b|$.

(OR)

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6 a) If a is a regular element of a semi group S then prove that every element of $D_u$ is regular.

b) Let Ø:S→T be a morphism from regular semi group S into na semi group T then im Ø is regular.If f is an idempotent in im Ø then there exists an idempotent e in S such that eØ=f.

UNIT-IV

7 a) If M is O – minimal ideal of S then prove that either $M^2 = \{0\}$ or M is a O – simple semigroup.

b) Prove that S is a completely O- simple semigroup.

(OR)

8 a) If S is a completely O- simple, then prove that S is a regular.

b) If S is a regular semigroup without zero in which every idempotent is primitive, then prove that S is completely simple.

UNIT-V

9 a) prove that $N_\rho$ is a normal sub group of G.

b) If the elements (a, i, λ) and ( b, j, μ) of S are such that (a, i, λ) ρ( b, j, μ) then prove that (λ,μ) ϵ $\rho_\Lambda$.

(OR)

10 a) Prove that the lattice of congruences on a completely 0- simple semigroup S is semi modular.

b) If S is a finite congruence- free semigroup without 0 and if |S| >2, then prove that S is a simple group.

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## M.Sc. MATHEMATICS-III SEMESTER
## M304(C)(21)(NR): NUMBER THEORY
## (With effect from the batch of students admitted during 2021-2022)

| | | | |
|---|---|---|---|
| Subject Code : | M304( C)(21) | I A Marks | 30 |
| No. of Lecture / Seminar/ Tutorial for week | 06L + 01 S/T | End Exam Marks | 70 |
| | | Total Marks | 100 |

**Course Objectives :** To develop problem solving skills and to acquire knowledge on basic concepts of Arithmetical Functions, Dirichlet Multiplication, Averages of Arithmetical Functions and Congruences.

**UNIT-I**

**ARITHMETICAL FUNCTIONS AND DIRICHLET MULTIPLICATION:** Introduction, The Mobius function $\mu(n)$, The Euler Totient function $\varphi(n)$, A relation connecting $\varphi$ and $\mu$, A product formula for $\varphi(n)$, The Dirichlet product of arithmetical functions, Dirichlet inverses and Mobius inversion formula.(Sections 2.1-2.7 of Ch 1)

**Learning outcomes**: Upon completion of this unit, the student will be able to:
Define and interpretthe concepts of divisibility, congruence, Dirichlet product.

**UNIT-II**

**ARITHMETICAL FUNCTIONS AND DIRICHLET MULTIPLICATION:**
The Mangoldt function $\Lambda(n)$, Multiplicative functions, Multiplicative functions and Dirichlet multiplication, The inverse of a completely multiplicative function, Liouville's function $\lambda(n)$, The divisor function $\sigma_z(n)$. Generalised convolutions. (Sections 2.8-2.14 of Ch 1)

**Learning outcomes**: Upon completion of this unit, the student will be able to:
Define and interpretthe concept multiplicative functions and Generalised convolutions.

**UNIT-III**

**AVERAGES OF ARITHMETICAL FUNCTIONS:**
Introduction, The big oh notation Asymptotic equality of functions, Euler's summation formula, Some elementary asymptotic formulas, The average order of d(n), The average order of divisor functions $\sigma_z(n)$,The average order of $\varphi(n)$, An application to the distribution of lattice points visible from the origin, Theaverage order of $\mu(n)$ and $\Lambda(n)$, The partial sums of a Dirichlet product, Applications to $\mu(n)$ and $\Lambda(n)$, Another identity for the partial sums of a Dirichlet product.

**Learning outcomes**: Upon completion of this unit, the student will be able to:
Understand the concepts of averages of arithmetical functions, prove and apply properties of multiplicative functionssuch as the Euler's phi function and of residues modulo n.

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**UNIT-IV**

**SOME ELEMENTARY THEOREMS ON THE DISTRIBUTION OF PRIME NUMBERS:**

Introduction, Chebyshev's functions $\psi(x)$and **I**(x). Relations connecting $\psi(x)$ and **I**(x), Some equivalent forms of the prime number theorem, Inequalities of $\pi(n)$ and $p_n$, Shapiro's Tauberian theorem, Application of Shapiro's theorem, An asymptotic formulae for the partial sums (1/p).

**Learning outcomes**: Upon completion of this unit, the student will be able to: Understand Chebyshev's functions $\psi(x)$and I(x) and the Relations connecting I(x) and $\pi(x)$, Some equivalentforms of the prime number theorem, Inequalities of $\pi(n)$ and pn, to study some applications of Shapiro's Tauberian theorem.

**UNIT-V**

**CONGRUENCES:** Definition and basic properties of congruences, Residue classes and complete residue systems, Linear congruences, Reduced residue systems and Euler - Fermat theorem, Polynomial congruences modulo p, Lagrange's theorem, Simultaneous linear congruences, The Chinese remainder theorem, Applications of the Chinese remainder theorem, Polynomial congruenceswith prime power moduli.

**Learning outcomes**: Upon completion of this unit, the student will be able to: Solve congruences of various types and use the theory of congruences in applications.

**PRESCRIBED BOOK: "Introduction to Analytic Number Theory",** Tom M.Apostol, NarosaPublishing House, New Delhi.

**Course outcomes:** After completing this course the student able to: Understand the properties of divisibility and prime numbers, compute the greatest common divisor and least common multiples,operations with congruences and use the Lagrane theorem, Fermat's theorem, Chinese remainder theorem.

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CODE: M304(C)(21)

# M.Sc. DEGREE EXAMINATION, MARCH 202__

## THIRED SEMESTER

## MATHEMATICS

## Paper –IV, NUMBER THEORY

## Model paper

Time: Three hours — Maximum : 70 marks

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Answer ONE question from each Unit — (5 x 14 = 70 marks)

### UNIT-1

1 (a) If n ≥ 1 then show that $\sum_{d/n} \varphi(d) = n$.

(b) for n ≥ 1 ,prove that $\varphi(n) = \prod_{\frac{p}{n}} \left(1 - \frac{1}{p}\right)$

(OR)

2 (a) Prove that Dirichlet multiplication is commutative and associative.

(b)Prove that for all f we have I * f = f * I =f.

### UNIT –II

3 (a) If both g and f * g are multiplicative , then f is also multiplicative.

(b) If g is multiplicative, so is $g^{-1}$, its Dirichlet inverse.

(OR)

4 (a) Let f be multiplicative .Then f is completely multiplicative

If and only if $f^{-1}(n) = \mu(n)\, f(n)$ for all n≥ 1

(b) Associative property relating ∘ and ∗. For any arithmetical Functions $\alpha$ and $\beta$ we have

$\alpha \circ (\beta \circ F) = (\alpha * \beta) \circ F$.

### UNIT-III

5 a) If x ≥ 1 then prove the following

(a) $\sum_{n \le x} \frac{1}{n} = \log x + c + 0\left(\frac{1}{x}\right)$.

(b) $\sum_{n \le x} \frac{1}{n^2} = \frac{x^{1-s}}{1-s} + \zeta(s) + 0(x^{-s})$ if s > 0, s ≠ 1.

(c) $\sum_{n > x} \frac{1}{n^2} = 0(x^{-s})$ if s > 1.

(d) $\sum_{n \le x} n^{\alpha} = \frac{x^{\alpha+1}}{\alpha+1} + 0(x^{\alpha})$ if $\alpha \ge 0$.

(OR)

6(a) For $x \geq 1$ ,show that

(i) $\sum_{n\leq x} \mu(n) \left[\frac{x}{n}\right] = 1$ and

(ii) $\sum_{n\leq x} \Lambda(n)\left[\frac{x}{n}\right] = \log\,[x]!$.

(b) If $x \geq 2$ then prove that $\log[x]\,! = x \log x - x + 0(\log x)$.

## UNIT-IV

7 a) Let $P_n$ denote the nth prime. Then the following asymptotic relations are logically equivalent :

(i) $\lim_{x\to\infty} \frac{\pi(x)\log x}{x} = 1.$

(ii) $\lim_{x\to\infty} \frac{\pi(x)\log \pi(x)}{x} = 1.$

(iii) $\lim_{n\to\infty} \frac{P_n}{n \log n} = 1.$

(OR)

8 a) Prove that there is a constant A such that

$\sum_{p\leq x} \frac{1}{p} = \log\log x + A + 0\left(\frac{1}{\log x}\right)$ for all $x \geq 2$.

## UNIT-V

9 a) Prove that a finite abelian group G of order n has exactly n distinct characters.

(OR)

10 a) Prove that there are infinitely many primes of the form 4n + 1 .

b) If $k > 0$ and $(h, k) = 1$,then prove that there are infinitely many primes in the arithmetic progression $nk+h$ , $n = 0,1,2,\ldots\ldots\ldots$

## M.Sc. MATHEMATICS-III SEMESTER
## M305(A)(21): MATHEMATICAL BILOGOY
### (With effect from the batch of students admitted during 2021-2022)

| | | | |
|---|---|---|---|
| Subject Code : | M305(A)(21) | I A Marks | 30 |
| No. of Lecture / Seminar/Tutorial for week | 06L + 01 S/T | End Exam Marks | 70 |
| | | Total Marks | 100 |

**Objective of the Course:**

1.This course is aimed to be accessible both to Master's students of biology who have a good understanding of the introductory course to mathematical biology and to Master's students in Mathematics looking to broaden their application areas.
2. The course extends the range of usage of mathematical models in biology, ecology and evolution.
3. Biologically, the course looks at models in evolution, population genetics and biological invasions.
4. Mathematically the course involves the application of multivariable calculus, ordinary differential equations and partial differential equations.
5. Formulation and analysis of ordinary differential equation (ODE) models for the population of a single species, finding equilibrium populations and determining how their stability depends on parameters.

**Unit I:**

Autonomous differential equations - Equilibrium solutions - Stability nature of equilibrium solutions, single species growth models involving exponential, logistic and Gompertz growths. Harvest models – bifurcations and break points. (Sections 1 and 2 of the Text Book).

**Learning out comes:** Upon Completing this unit, students will be able to

1. Understand the concept of single species growth models involving exponential logistic and Gompertz growths. Harvest models, bifurcations and break points.
2. Convert verbal descriptions of biological systems into appropriate mathematical models amenable to quantitative and qualitative analysis.
3. Develop the ability to explain mathematical results in language understandable by biologists.

**Unit II:**

LotkaVolterra predator – prey model – phase plane analysis, General predator prey systems – equilibrium solutions – classification of equilibria – existence of cycles – Bendixson-Dulac's negative criterion – functional responses. (Sections 7 and 8 of the text book).

**Learning out comes**: Upon completing this unit students will be able to

1. Understand the concept of Lotka Volterra predator, pry model, phase palne analysis and applications of Bendixson-Dulac's negative criterion.
2. Identify the equilibrium points and study the phase portrait analysis of predator prey model.
3. Perform elementary mathematical analysis of models introduced and interpret conditions obtained from the analysis, usually taking the form of relationships between model parameters-that correspond to specific model behavior, and express the ramifications for the biological process being considered.

**Unit III:**

Global bifurcations in predator prey models – Freedman and Wolkowicz model - type IV functional response – Hopf bifurcation – Homoclinic orbits – Global bifurcations using Allee effect in prey – Competition models – (Sections 9 and 10 of the text book).

**Learning out comes:** Upon completing this unit, students will be able to

1. Understand the concept of Global bifurcations in predator prey model, Gobal bifurcations using Allee effect in prey – competition models and applications.
2. Understand and apply the concept of stability of a fixed point solution of a system of ordinary differential equations.
3. Analyze the model with graphical representation and give biological interpretation.

**Unit IV:**

Lotka – Voltrrra Competition model – exploitation competition models. Mutualism models – various types of mutualisms – cooperative systems – Harvest models and optimal control theory (Sections 11 and 12 of the text book).

**Learning out comes:** Upon competing this unit, students will be able to

1. Analyze ODE models for the populations of two interacting species.
2. Identify equilibrium points and using information about their linear stability to characterize the long-term behavior of the system.
3. Analyze the model with graphical representation and give biological interpretation.

**Unit V:**

Open access fishery – sole owner fishery – Pontryagin's maximum principle – Economic interpretation of Hamiltonian and adjoint variable. (Sections 13 and 14 of the text book)

**Learning out comes:** Upon completing this unit, students will be able to

1. Understand the concepts of open access fishery, sole owner fishery.
2. Apply Pontryagin's maximum principle to open access fishery, sole owner fishery.
3. Analyze economical interpretation for sole owner fishery.

**Text book:** Mark Kot, 2001, Elements of Mathematical Ecology, Cambridge University Press.
Reference: Nisbet and Gurney, 1982, Modeling Fluctuating Populations, John Wiley & Sons.

**Course out comes:** On successful completion of this course the students will be able to

1. Read, situate, and understand research papers in the area of mathematical biology.
2. Prepare to discuss specific biological systems with life scientists, and in particular communicate efficiently how values of model parameters can impact the qualitative behavior of the system..
3. Solve mathematically and interpret biologically simple problems involving one and two species ecosystems, epidemics and biochemical reactions.
4. Analyze the model with graphical representation and give biological interpretation for competition models, mutualism models.
5. Analyze economical interpretation for open access fishery, solve owner fishery models using pontryagin's maximum principle.

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# M.Sc. DEGREE EXAMINATION, MARCH 202__
## THIRED SEMESTER
## MATHEMATICS
## Paper –V, MATHEMATICAL BIOLOGY
## Model paper

Time: Three hours | Maximum : 70 marks

Answer ONE question from each Unit (5 x 14 = 70 marks)

### UNIT I

1(a) Let $x^*$ be an equilibrium point of $x^1 = f(x)$. Discuss the stability nature of $x^*$.

(b) Consider the logistic differential equation subjected to harvesting with catch per unit effort proportional to the stock and analyze it by taking effort as a bifurcation parameter.

(OR)

2(a) Explain Gompertz equation and discuss qualitative behavior of solutions of Gompertz equation.

(b) Illustrate four different one-parameter bifurcations throught appropriate examples.

### UNIT-II

3(a) Show that the Lotka-Volterra model admits a non-trivial stable equilibrium point which is surrounded by a family of periodic orbits.

(b) Analyze the Lotka-Volterra model through phase plane analysis and discuss the possible scenarios for its interior equilibrium point with respect to variations in the involved parameters.

(OR)

4(a) Classify equilibria of a general predator-prey system through community matrix.

(b) Describe the qualitative behavior of solutions of Rosenzweig-Mac Arthur model.

### UNIT-III

5. Discuss the local and global bifurcations for the following model

$$\frac{dN}{dT} = rN\left(1-\frac{N}{K}\right) - \phi(N)P$$

$$\frac{dP}{dT} = b\phi(N)P - mP, \ \ where\ \phi(N) = \frac{cN}{\dfrac{N^2}{i} + N + a}.$$

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(OR)

6. Discuss the global bifurcations with Allee effect in the prey.

UNIT-IV

7.(a) Derive Lotka-Volterra Competition model and analyze it.

(b) Discuss the qualitative behavior of solutions of the equation

$$\frac{dy}{dx} = \frac{\delta h(x)y}{f(x)[g(x)-y]}$$

With $\delta$ as bifurcation parameter.

(OR)

8(a) Explain different types of mutualisms.

(b) Define cooperative system. Give and example. Prove that the orbits of a cooperative system either converge to equilibria or diverge to infinity.

UNIT-V

9. Discuss the economic aspects of open-access fishery by using dynamic approach.

(OR)

10. Solve the problem of sole-owner fishery with discounting.

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**M.Sc. MATHEMATICS III SEMESTER**
**M 305B(21)(NR) - LINEAR PROGRAMMING**
**(With effect from the batch of students admitted during 2021-2022)**

| Subject Code : | M305B(21) | I A Marks | 30 |
|---|---|---|---|
| No. of Lecture / Seminar/ Tutorial for week | 06L + 01 S/T | End Exam Marks | 70 |
| | | Total Marks | 100 |

**Course Objectives :** To develop problem solving skills and to acquire knowledge on basic concepts of in linear programming problems, Transportation problems, Assignment problems and Job sequencing.

**UNIT – I**

Mathematical Back ground: Lines and hyper planes: Convex sets, convex sets and Hyper planes, convex cones. (Sections 2.19 to 2.22 of Chapter 2of [1]).

**Theory of the simplex method :** restatement of the problem, slack and surplus Variables , reduction of any feasible solution to a basic feasible solution , some definitions and notations ,improving a basic feasible solution, unbounded solutions, optimality conditions alternative optima , Extreme points and basic feasible solutions. (Sections 3.1, 3.2, 3.4 to 3.10 of Chapter 3 of [1])

**Learning outcomes**: Upon completion of this unit, the student will be able to: Formulate and solve a linear programming problem.

**UNIT –II**

Detailed development and Computational aspects of the simplex method, The Simplex method, selection of the vector to enter the basis, degeneracy and breaking ties further development of the transportation formulas, the initial basic feasible solution –artificial variables, Tableau format for simplex computations, use of the tableau format, conversion of a minimization problem to a maximization problem, Review of the simplex method, illustrative examples. (Sections 4.1 to 4.5 & 4.7 to 4.11 of Chapter 4 of [1]).

**Learning outcomes**: Upon completion of this unit, the student will be able to: Convert standard business problems into linear programming problems and can solve using simplex algorithm.

**UNIT –III**

**Transportation problems:** Introduction, properties of the matrix A: the simplex Method and transportation problems, simplifications resulting from all $y_{ij}\alpha\beta = \pm$ 1or 0, The Stepping-Stone algorithm.(Sections 9.1 to 9.7 of Chapter 9of [1] ).

**Learning outcomes**: Upon completion of this unit, the student will be able to: Formulate and solve transportation problems.

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**UNIT –IV**

Determination of an initial basic feasible solution, alternative procedure for computing $z_{ij} - c_{ij}$; duality ( Sections 9.10 & 9.11 of chapter 9 of [1] )

**Learning outcomes:** Upon completion of this unit, the student will be able to: Formulate and solve transportation problems by using the Stepping – Stone algorithm.

**UNIT –V**

**The assignment problem:** Introduction, description and mathematical statement of the problem, Solution using the Hungarian method, the relationship between transportation and assignment problems, further treatment of the assignment problem, the bottle neck assignment problem. (Sections 6.1 to 6.6 of Chapter-6 of [2] )

**Learning outcomes:** Upon completion of this unit, the student will be able to: Formulate and solve the Assignment problem.

**TEXT BOOK:**

[1] **G.Hadley** "Linear Programming" Addison-Wesley Publishing Company.

[2] **Benjamin Lev and Howard J.Weiss** "Introduction to Mathematical Programming" Edward Arnold Pub, London, 1982.

**Course outcomes:** After completing this course, the student acquaints him (her) self in the mathematical methods for solving transportation problem and assignment problem.

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CODE: M305(B)(21)(NR)

M.Sc. DEGREE EXAMINATION, MARCH 202__

THIRED SEMESTER

MATHEMATICS

Paper – V, LINEAR PROGRAMMING

Model paper

Time: Three hours — Maximum : 70 marks

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Answer ONE question from each Unit (5 x 14 = 70 marks)

**UNIT I**

1. (a) Define convex set. Prove that the collection of all feasible solutions to a L.P.P constitutes a convex set whose extreme points correspond to the Basic feasible solution.

(b) Prove that the set $X=\{[x,x_2]/x_1^2+,x_2^2\leq 1\}$ is convex.

**(OR)**

2. Find all basic feasible solutions for the system of equations

$2x_1+6x_2+2x_3+x_4 = 3$

$6x_1+4x_2+4x_3+6x_4 = 2,\ x_1,x_2,x_3,x_4 \geq 0.$

**UNIT – II**

3. Solve the following L.P.P using artificial variable technique

Min$z = 3x_1+44x_2+x_3+6x_4$

Subject to $5x_1-2x_2-x_3+3x_4 \geq 2,$

$6x_1+x_2-5x_3-3x_4 \geq 5$

$-x_1+4x_2+3x_3+7x_4 \geq 6$, and $x_1,x_2,x_3,x_4 \geq 0.$

**(OR)**

4. Solve the following L.P.P using simplex method.

Max $z = 6x_1 - 2x_2$

*Subjected to* $2x_1 - x_2 \leq 2$

$x_1 \leq 4$, and $x_1, x_2 \geq 0.$

**UNIT – III**

5. Find the optimal solution by Stepping stone method to find the Initial basic feasible solution using the Row minima method for the following transportation problem

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| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
|---|---|---|---|---|---|
| $S_1$ | 11 | 13 | 17 | 14 | 250 |
| $S_2$ | 16 | 18 | 14 | 10 | 300 |
| $S_3$ | 21 | 24 | 13 | 10 | 400 |
| Demand | 200 | 225 | 275 | 250 | |

**(OR)**

6. Solve the following Transportation Problem using Stepping Stone algorithm.

| | I | II | III | IV | supply |
|---|---|---|---|---|---|
| A | 40 | 44 | 48 | 35 | 160 |
| B | 37 | 45 | 50 | 52 | 150 |
| C | 35 | 40 | 45 | 50 | 190 |
| Demand | 80 | 90 | 110 | 220 | |

## UNIT-IV

7. Find the optimal solution by MODI method to find the Initial basic feasible solution using the Vogel's method for the following transportation problem

| Origins | I | II | III | Available |
|---|---|---|---|---|
| A | 2 | 7 | 4 | 5 |
| B | 3 | 3 | 1 | 8 |
| C | 5 | 4 | 7 | 7 |
| D | 1 | 6 | 2 | 14 |
| Requirement | 7 | 9 | 18 | |

**OR**

**8.** Explain Matrix minima method .Find the initial basic feasible solution by matrix minima method for the the following transportation problem

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| | I | II | III | IV | Supply |
|---|---|---|---|---|---|
| A | 4 | 6 | 8 | 13 | 550 |
| B | 13 | 11 | 10 | 8 | 700 |
| C | 14 | 4 | 10 | 13 | 300 |
| D | 9 | 11 | 13 | 3 | 500 |
| Demand | 250 | 350 | 1050 | 200 | |

**UNIT - V**

9. Define (i) balanced (ii) unbalanced assignment problems and solve the following assignment problem by using Hungarian method.

Machine

Jobs

| | I | II | III | IV | V |
|---|---|---|---|---|---|
| A | 45 | 30 | 65 | 40 | 55 |
| B | 50 | 30 | 25 | 60 | 30 |
| C | 25 | 20 | 15 | 20 | 40 |
| D | 35 | 25 | 30 | 30 | 20 |
| E | 80 | 60 | 60 | 70 | 50 |

**(OR)**

10. (a) Solve the following by using bottle neck assignment algorithm.

Workers

Jobs

| | A | B | C | D |
|---|---|---|---|---|
| 1 | 2 | 4 | 2 | 4 |
| 2 | 8 | 5 | 4 | 5 |
| 3 | 4 | 6 | 8 | 9 |
| 4 | 8 | 4 | 2 | 4 |

(b) Explain the difference between transportation and assignment problems

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## M.Sc. MATHEMATICS III SEMESTER
## M 305(C)B(21) - MATHEMATICAL METHODS
### (With effect from the batch of students admitted during 2021-2022)

| Subject Code : | M305(C)(21) | I A Marks | 30 |
|---|---|---|---|
| No. of Lecture / Seminar/ Tutorial for week | 06L + 01 S/T | End Exam Marks | 70 |
| | | Total Marks | 100 |

**UNIT-I:**

Integral Equations: Relation between differential and integral equations, Solutions of integral equations, Volterra integral equations of first and second kind, iterated kernals, Fredholm integral equations, Neumann series, Recciprocal Kernals, Degenerate kernals, Hilbert Schmidt theory, Symmetric kernals with eigen values, iterative methods for solving equations of the second kind, Singular integral equations. [Chapter 2 of Ref(2) and sections 3.2, 3.6 to 3.12 of chapter 3 of Ref(1)]

**UNIT-II:**

Application of Integral equations: Initial and boundary value problems, longitudinal vibrations and deformation of a rod, Green's function, Construction of Green's functions[chapter 5 of Ref(2)]

**UNIT-III:**

The Calculus of variations: Euler's Equation – functions of the form $\int_{x_2}^{x_1} f(x, y_1, y_2, \dots y_n, y_1', y_2', \dots, y_n')dx$ -functional dependence on higher order derivatives-Variational problems in parametric form and applications. [Chapter VI of Ref-2]

**UNIT-IV:**

Tensor Analysis: N- dimensional space – Covariant and Contravariant vectors – Contraction –Second and higher order tensors– Quotient law–Fundamental tensor [Chapter.1,2 of Ref 4]

**UNIT-V:**

Associate temsor – Angle between the vectors – Principal directions – Christoffel symbols – Covariant and intrinsic derivatives – Geodesics.[Chapter 3,4 of Ref Text Book 4]

**References:**


1. Methods of Applied Mathematics by Francis B.Hildband, Printice Hall of India, Second Edition(1972).
2. Integral Equations by Shanti Swarup, Krishna Prakasan Media(p) Ltd., Meerut.
3. Differential Equations by L.Elsgolts, Mir Publishers, Moscow.
4. Tensor Calculus by Barry Spain.


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CODE: M305(C)(21)

# M.Sc. DEGREE EXAMINATION, MARCH 202_
## THIRED SEMESTER
## MATHEMATICS
## Paper –V, MATHEMATICAL METHODS

**Model paper**

Time: Three hours | Maximum : 70 marks

Answer ONE question from each Unit (5 x 14 = 70 marks)

**Unit-1**

1. A) With the help o the resolvent kernel, obtain the solution of the integral equation:

$$\phi(x) = x + \int_0^x (\xi - x)\phi(\xi)d\xi .$$

B) Show that the function $\phi(x) = xe^x$ is a solution of the Volterra integral equation: $\phi(x) = \sin x + 2\int_0^x \cos(x-\xi)\phi(\xi)d\xi.$

(OR)

2. C) Solve the integral equation $\phi(\xi) = \sin\xi + \int_0^\pi \cos\xi \sin x \phi(x)\,dx.$

D) Explain the method of generating the sequences of eigen values and eigen functions for the integral equation $\phi(x) = \lambda\int_a^b K(\xi, x)\phi(\xi)\,d\xi$

**Unit-2**

3. Transform the differential equation, $x^2\dfrac{d^2u}{dx^2} + x\dfrac{du}{dx} + (\lambda x^2 - 1)u = 0$ with the boundary conditions $u(0) = u(1) = 0$ into an integral equation, by constructing Green's function.

(OR)

4. Solve the boundary value problem using Green's function $\dfrac{d^2u}{dx^2} - u = -2e^x$ with $u(0) = u'(0), u(l) + u'(l) = 0$

**Unit-3**

5. A) Obtain a necessary condition for the extremals of the functional

$$v(y(x)) = \int_{x_0}^{x_1} F(x; y(x); y'(x))\,dx \text{ satisfying } y(x_0) = y_0; y(x_1) = y_1.$$

B. So...

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B) Find the extremals of the functional $v(y(x))=\int_{x_0}^{x_1}\left[(y''')^2+y^2-2yx^3\right]dx$

(OR)

6. C) Determine a curve joining the origin with the point A(1,1) whose rotation about the axis of abscissa generates a surface of minimum area.

D) Determine the differential equation satisfied by the free vibrations of a string.

Unit-4

7. A)State and prove Quotient law of tensors.

B) If $A^i$ is an arbitrary contravariant vector and $c_{ij}A^iA^j$ is an invariant, Show that $(c_{ij}+c_{ji})$ is a covariant tensor of second order.

(OR)

8. C) A covariant tensor of first order has components $xy, 2y-z^2, xz$ in rectangular coordinates. Determine its covariant components in spherical polar coordinates.

D) Prove that fundamental tensor $g_{ij}$ is a covariant symmetric tensor of order two.

Unit-5

9. A) Obtain the christoffel symbols corresponding to the metric $ds^2=(dx^1)^2+G(x^1,x^2)(dx^2)^2$ where G is a function of $x^1$ and $x^2$.

B) Prove that $div\ \mathrm{A}_j=\frac{1}{\sqrt{g}}\frac{\partial}{\partial x^r}\{\sqrt{g}\,g^{rk}A_k\}=div\ \mathrm{A}^j$.

(OR)

10. C) Define a geodesic and a null-geodesic. Obtain the N differential equations of a geodesic of the second order.

D) Explain the concept o parallelism along a curve. Show that the vector obtained by the parallel propagation of the tangent vector to a geodesic always remains tangent to the geodesic.

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# M.Sc. MATHEMATICS-I SEMESTER
## M401 (21)(NR): NON COMMUTATIVE RINGS
(With effect from the batch of students admitted during 2021-2022)

| | | | |
|---|---|---|---|
| Subject Code : | M401 (21) | I A Marks | 30 |
| No. of Lecture / Seminar/ Tutorial for week | 06L + 01 S/T | End Exam Marks | 70 |
| | | Total Marks | 100 |

**UNIT –I**

Primitive Rings, Radicals, completely reducible modules. (Sections 3.1 to 3.3 of Chapter 3)

**UNIT – II**

Completely reducible rings, Artinian and Noetherian rings, On lifting idempotents,. (Sections 3.4, to 3.6 of Chapter 3)

**UNIT – III**

Local and semiperfect rings, Projective modules, Injective modules.(Section 3.7 of Chapter 3 & Sections 4.1 to 4.2 of Chapter 4)

**UNIT –IV**

The complete ring of quotients, Rings of endomorphism's of injective modules. (Sections 4.3 to 4.4 of Chapter 4)

**UNIT –V**

Tensor products of modules, Hom and functors exact sequences.(Sections 5.1 to 5.3 of Chapter 5)

**TEXT BOOK:**

J. Lambek **"Lectures on Rings and Modules"** A Blaisdell book in Pure and Applied Mathematics.

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CODE: M401(21)(NR)

# M.Sc. DEGREE EXAMINATION, MARCH 202__
## FOURTH SEMESTER
## MATHEMATICS
## Paper I, NON-COMMUTATIVE RINGS
### Model paper

Time: Three hours                                                                 Maximum : 70 marks

Answer ONE question from each Unit                                                  (5 x 14 = 70 marks)

## UNIT I

1. (a) Show that every primitive ideal is prime.

   (b) Define Prime radical of a ring R. Show that the radical of R is the set of all $r \in R$ such that 1- rs is right invertible for all $s \in R$.

**(OR)**

2. Define a completely reducible module, socle of a module. Show that the following conditions concerning a module $A_R$ are equivalent:

   (i) A = Soc A.

   (ii) A is the sum of minimal submodules.

   (iii) A is isomorphic to a direct sum of irreducible modules.

## UNIT II

3 (a) If R is semiprime and $e^2 = e \in R$, then show that eR is minimal right ideal if and only if eRe is division ring.

  (b) If R is right Artinian then show that Rad R = rad R.

(OR)

4 (a) Show that in a regular ring every finitely generated right ideal is principal.

  (b) ) Show that in any ring idempotents modulo the prime radical can be lifted.

## UNIT III

5 (a) Define a local ring. Show that a ring R is local if and only if it is semi perfect and 1 is a primitive idempotent

  (b) Let e is an idempotent of R and N = Rad R then show that Rad (eRe) = eRe $\cap$ N = eNe

(OR)

6(a). Define a projective module. If M is a direct sum of a family of modules $\{M_i / i \in I\}$ then prove that M is projective if and only if each $M_i$ is projective.

(b) Define an Injective module. Prove that every module is isomorphic to a submodule of the character module of a free module

## UNIT IV

7(a). Show that an ideal D of a ring R is dense as a right R- module if and only if

$r_1 D = 0 \Rightarrow r_1 = 0 \ \forall \ r_1 \in R$

(b) Let R be a prime ring with non zero socle .Then show that Q is the ring of all linear transformations of a vector space.

(OR)

8(a). Assume that every non zero submodule of $M_R$ is large. Then prove that its injective hull $I_R$ is indecomposable and H is a local ring.

(b) If the injective hull $I_R$ is injective and indecomposable then prove that $Hom_R(I,I)$ is a local ring.

## UNIT V

9(a). If $A_R$ , $_RB$ be right and left R-modules. Then prove that $A \otimes_R B$ is an abelian group under addition.

(b) Let $\Phi$ be a bilinear mapping from ($A_R$, $_RB$) to C.Then prove that there exists a unique homomorphism $\phi$: $A \otimes_R B \to C$ such that $\pi \circ \phi = \Phi$.

(OR)

10(a). Prove that the pair $A \xrightarrow{f} B \xrightarrow{g} C$ is exact if and only if the pair $C^* \xrightarrow{g^*} B^* \xrightarrow{f^*} A$ is exact.

(b) For any R-module A, prove that there exists a free module F such that $F \to A \to 0$ is exact.

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## M.Sc. MATHEMATICS-IV SEMESTER

## M402 (21)(NR): PARTIAL DIFFERENTIAL EQUATIONS

**(With effect from the batch of students admitted during 2021-2022)**

| Subject Code : | M402(21) | I A Marks | 30 |
|---|---|---|---|
| No. of Lecture / Seminar/ Tutorial for week | 06L + 01 S/T | End Exam Marks | 70 |
| | | Total Marks | 100 |

**Course Objectives:** To introduce first and second order partial differential equations and their classifications and methods of finding solutions of these partial differential equations.

**UNIT-I**

**First Order Partial Differential equations**: Curves and Surfaces - Genesis of first order partial differential equations - Classification of integrals - linear equations of the first orderPartial Differential equations - Compatible systems. (Sections 1.1 to 1.6 of Chapter 1 of [1])

**Learning outcomes**: Upon completion of this unit, the student will be able to: Classify first order partial differential equations and their solutions and solve them using some methods.

**UNIT-II**

Charpit's method – Jacobi's method - Integral surfaces through a given curve- **Second order Partial differential Equations**: Genesis of Second Order Partial Differential Equations - Classification of Second Order Partial differential equations.
(Sections 1.7 to 1.9 of Chapter 1 and Sections 2.1 to 2.2 of Chapter 2 of [1]).

**Learning outcomes**: Upon completion of this unit, the student will be able to: apply Charpit's and Jocobi's methods to solve first order partial differential equations and Classifyand solve second order partial differential equations.

**UNIT-III**

**One Dimensional Waves equations**: Vibrations of an infinite string - Vibrations of a semi-infinite string - Vibrations of a string of Finite Length - Riemann's Method - Vibrations of a string of finite length (method of separation of variables) - **Laplaces Equation**: Boundary value problems - Maximum and minimum principles.
(Sections 2.3.1 to 2.3.5 of Chapter 2 and Sections 2.4.1to 2.4.2 of Chapter 2 of [1]).

**Learning outcomes**: Upon completion of this unit, the student will be able to: solve one dimensional wave equations using different analytic methods and understand Laplace equationsand Maximum and minimum principles.

B. Satya (signature) R. Srinu (signature)

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**UNIT-IV**

The Cauchy problem - The Dirichlet problem for the upper Half plane - The Neumann problem for the upper Half plane - the Dirichlet problem for a circle - the Dirichlet Exterior problem for a circle –TheNeumann problem for a circle- The Dirichlet problem for a Rectangle – Harnack's Theorem.
(Sections 2.4.3 to 2.4.10 of Chapter 2 if [1])

**Learning outcomes:** Upon completion of this unit, the student will be able to: Solve Laplace equations using various analytical methods demonstrate uniqueness of solutions of certain kinds of these equations

**UNIT-V**

Laplace's Equation – Green's Function- The Dirichlet problem for a Half plane -The Dirichlet problem for a circle - Heat conductioninfinite rod case - Heat conduction Finite rod case -**Duhamel's principle**: Wave equation - Heat conduction equation.
(Sections 2.4.11 to 2.4.13 and 2.5.1 to 2.5.2 and 2.6.1 to 2.6.2 of Chapter 2 of [1])

**Learning outcomes:** Upon completion of this unit, the student will be able to: Compute solutions of heat equations using certain analytic methods and verify uniqueness of solutions of some types of these equations.

**TEXT BOOK:** An Elementary course in Partial Differential Equations by T.Amaranath, Published by Narosa Publishing House.

R. Srinu

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CODE: M402 (21)

# M.Sc. DEGREE EXAMINATION,
# FOURTH SEMESTER
# MATHEMATICS
# Paper –II, PARTIAL DIFFERENTIAL EQUATIONS
## Model paper

Time: Three hours | Maximum : 70 marks

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Answer ONE question from each Unit (5 x 14 = 70 marks)

## UNIT I

1. (a) Find the general integral of (y + 1) p + (x + 1) q = z.

   (b) Show that a necessary and sufficient condition that the equation Pdx + Qdy + Rdz = 0, be integrable is that $\left(\vec{X}.\text{curl}\,\vec{X}\right) = 0$, where $\vec{X} = (P, Q, R)$.

Or

2. (a) Verify that the Pfaffian differential equation

$$(y^2 + yz)dx + (xz + z^2)dy + (y^2 - xy)dz = 0$$

   is integrable and find its integral.

   (b) Show that the equations $f = xp - yq - x = 0$

$$g = x^2p + q - xz = 0$$

   are compatible and find a one–parameter family of common solutions.

## UNIT II

3. (a) Describe the Charpit's method to find a complete integral of a first order PDE.

   (b) Find a complete integral of $z^2 - pqxy = 0$ by Charpit's method.

Or

4. (a) Find a complete integral of the equation $xp^2 + yq^2 = z$ by Jacobi's method.

   (b) Reduce the equation $u_{xx} - x^2u_{yy} = 0$ toa canonical form.

R. Srinu

B. Sat

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UNIT III

5. (a) Deduce the d'Alembert's solution of the IVP of vibrations of an infinite string.

(b) Deduce the solution of the IVP of Vibration of semi – infinite string using d'Alembert's solution..

Or

6. (a) Solve the IVP/BVP of vibrations of a string of finite length by method of separation of variables.

(b) State and prove maximum principle.

UNIT IV

7. Show that the solution of the Dirichlet problem for a circle of radius a is given by thePoisson integral formula.

8. (a) Solve the IVP/BVP of the Dirichlet problem for a rectangle.

(b) State and prove Harnack's theorem.

UNIT V

9. (a) Solve the Dirichlet problem for half plane usingGreen's function.

(b) Solve the IVP/BVP of heat conduction in a finite rod of length $\ell$, using method of separation of variables.A

.Or

10. (a) Solve using Duhamel's principle the non-homogeneous wave equation $u_{tt} - c^2u_{xx} = F(x, t)$, $-\infty < x < \infty$, $0 < t$ with homogeneous initial conditions $u(x, 0) = u_t(x, 0) = 0$, $-\infty < x < \infty$.

(b) Solve the IVP/BVP problem of heat conduction in an infinite rod.

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**DEPARTMENT OF MATHEMATICS** **M 403 (NR)**

**ACHARYA NAGARJUNA UNIVERSITY**

**M.Sc., Mathematics**

**SYLLABUS**

**(With effect from the batch of students admitted during 2021-2022)**

**IV-SEMESTER**

**M 403 – NEAR-RINGS**

| | | | |
|---|---|---|---|
| Subject Code: | M 403 | I A Marks | 30 |
| No. of Lecture / Seminar / Tutorial for week | 06L + 01 S/T | End Exam Marks | 70 |
| | | Total Marks | 100 |

**Course Objectives:** To introduce Near-rings and its basic concepts and structure theory namely, subnear-rings, ideals, homomorphisms, direct products, subdirect products, sums and direct sums of ideals, prime and semiprime ideals, structure theory and primitive near-rings.

**Unit I**

**The Elementary Theory of Near-Rings**

(a) Fundamental definitions and properties

1. Near-rings
2. N-groups
3. Substructures
4. Homomorphisms and ideal-like concepts
5. Annihilators
6. Generated objects.

**(Section (a) of Chapter – 1)**

**Learning outcomes**: Upon completion of this unit, the student will be able to understand near-rings, subnear-rings, ideals and homomorphisms and the results based on these concepts.

**Unit II**

(b) Constructions:

1. Products, direct sums and subdirect products

(c) Embeddings

1. Embeddings in $M(\Gamma)$

**Ideal Theory**

(a) Sums:

1. Sums and direct sums
2. Distributive sums

(b) Chain conditions

**(Sections (b) (1) & (c) (1) of Chapter – 1 and Sections (a) & (b) of Chapter – 2)**

**Learning outcomes:** Upon completion of this unit, the student will be able to know and understand direct products and sums of near-rings and also direct sums of ideals and chain conditions on ideals.

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**Unit III**

(c) Decomposition theorems

(d) Prime ideals

1. Products of subsets
2. Prime ideals
3. Semiprime ideals

**(Sections (c) & (d) of Chapter -2)**

**Learning outcomes**: Upon completion of this unit, the student will be able to understand decomposition theorems in near-rings and prime and semiprime ideals of near-rings.

**Unit IV**

(e) Nil and nilpotent

**Structure Theory:**

Elements of the structure theory

(a) Types of N-groups

(b) Change of the Near-ring

(c) Modularity

**(Section (e) of Chapter-2 and Sections (a), (b) & (c) of Chapter-3)**

**Learning outcomes**: Upon completion of this unit, the student will be able to understand and apply types of N-groups, modularity in near-rings.

**Unit V**

**Structure Theory:**

(d) Quasiregularity

**Primitive Near-Rings:**

(a) General

1. Definitions and elementary results
2. The centralizer
3. Independence and density

(b) 0-primitive near-rings

**(Section (d) of Chapter-3 and Sections (a) & (b) of Chapter-4 )**

**Learning outcomes**: Upon completion of this unit, the student will be able to understand quasiregularity in near-rings and structure of primitive near-rings.

**Prescribed Book:**

Near-Rings, The Theory and its Applications by Gunter Pilz, North-Holland Publishing Company, AMSTERDAM, Revised Edition, 1983.

KGprasad

B. Satyanarayana

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# M.Sc. DEGREE EXAMINATION

Fourth Semester

Mathematics

Paper III – NEAR–RINGS (MODEL PAPER)

Time: Three hours | Maximum : 70 marks.

Answer ONE question from each Unit.

## UNIT I

1. (a) State and prove homomorphism theorem for near-rings.

   (b) (i) Prove that $N_0 \trianglelefteq_l N$, but not generally $N_0 \trianglelefteq N$.

   (ii) Prove that $N_c$ is an invariant subnear-ring of $N$, but in general neither a right nor a left ideal.

**(OR)**

2 (a) Prove that $I \triangleleft N$ is maximal in $N$ if and only if $N/I$ is simple.

   (b) Prove that ${}_N\Gamma$ is faithful if and only if $N \subset_{>} M(\Gamma)$.

## UNIT II

3. Let $(I_i)_{i \in K}$ be a family of ideals of near-ring $N$. Prove that the following are equivalent.

   (i) The sum of $I_i$'s is direct

   (ii) The sum of the normal subgroups $(I_i, +)$ is direct

   (iii) $\forall i \in K : I_i \cap \left( \sum_{\substack{j \in K \\ j \neq i}} I_j \right) = \{0\}$.

**(OR)**

4. (a) Prove that if $I \trianglelefteq N$ is a direct summand then each ideal of $I$ is an ideal of $N$.

   (b) Prove that a sequence $N = N_0 \supset N_1 \supset N_2 \supset \ldots.. \supset N_n = \{0\}$ of subnear-rings $N_i$ of $N$ is a composition sequence $\Leftrightarrow$ all factors are simple.

B. Satish

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## UNIT III

5 Let $P$ be an ideal of a near-ring $N$. Prove that the following are equivalent.

(i) $P$ is a prime ideal.

(ii) $\forall I, J \trianglelefteq N: (IJ) \subseteq P \Rightarrow I \subseteq P \vee J \subseteq P.$

(iii) $\forall i, j \in N: i \notin P \wedge j \notin P \Rightarrow (i)(j) \not\subset P.$

(iv) $\forall I, J \trianglelefteq N: I \supset P \wedge J \supset P \Rightarrow IJ \not\subset P.$

(v) $\forall I, J \trianglelefteq N: I \not\subset P \wedge J \not\subset P \Rightarrow IJ \not\subset P.$

**(OR)**

6 (a) Let $(P_\alpha)_{\alpha \in A}$ be a family of prime ideals, totally ordered by inclusion. Then prove that $\bigcap_{\alpha \in A} P_\alpha =: P$ is a prime ideal too.

(b) Prove that $N$ is simple $\Rightarrow$ $N$ is prime or $N$ is a zero near-ring.

## UNIT IV

7 (a) Let $I \trianglelefteq N$. Then prove that $N$ is nil if and only if $I$ and $N/I$ are nil.

(b) Let ${}_N\Gamma$ be of type 0 (with generator $\gamma$) and let $L \leq_N N$ be a minimal left ideal with $L \not\subset (0:\gamma)$. Then prove that $L \cong_N \Gamma$.

**(OR)**

8 (a) Let $\Gamma$ be an $N-$group and $\Delta$ a subset of $\Gamma$. Prove the following:

(i) ${}_N\Gamma$ is faithful if and only if ${}_{N_0}\Gamma$ and ${}_{N_c}\Gamma$ are faithful

(ii) $\Delta \trianglelefteq_N \Gamma \Leftrightarrow \Delta \trianglelefteq_{N_0} \Gamma$

(iii) $\Delta \leq_N \Gamma \Leftrightarrow \Delta \leq_{N_0} \Gamma \wedge \Omega \subseteq \Delta.$

(b) Prove that each modular left ideal $L \neq N \in \eta_0$ is contained in a maximal one.

## UNIT V

9. (a) Let $N \in \eta_0$. Prove the following:

(i) $z \in N$ is nilpotent $\Rightarrow$ $z$ is quasiregular.

(ii) Each nil subset of $N$ is quasiregular

(iii) If $L \triangleleft_l N$ is modular by $e$, then $e$ is not quasiregular

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(b) Let $N$ contain either a left or a right identity $e$. Then prove the following:

(i) Every $\upsilon-$primitive ideal $I$ of $N$ is modular.

(ii) If $e$ is a left identity of $N$ then $N$ is $1-$primitive iff $N$ is $2-$primitive.

**(OR)**

10 (a) Let the ring $N$ be $\upsilon-$primitive on $\Gamma$. Then prove that $N$ is a primitive ring on the $N-$ module $\Gamma$.

(b) If ${}_N\Gamma$ is unitary and $N = N_0$ then prove that $\sim$ and $\approx$ coincide on $\theta_1$.

**M.Sc. MATHEMATICS –IV SEMESTER**
**M 404A(21)(NR) – ALGEBRAIC CODING THEORY**
**(With effect from the batch of students admitted during 2021-2022)**

| Subject Code : | M404(A)(21) | I A Marks | 30 |
|---|---|---|---|
| No. of Lecture / Seminar/Tutorial for week | 06L + 01 S/T | End Exam Marks | 70 |
| | | Total Marks | 100 |

**Course Objectives:** To detect errors in transmission of messages and to introduce the basic concepts of Coding theory such as linear codes, perfect and Related codes, and Cyclic Linearcodes.

**UNIT –I**

**Introduction to Coding Theory:** Introduction, Basic assumptions, Correcting and Detecting error patterns, Information Rate, The Effects of error Correction and Detection, Finding themost likely codeword transmitted, Some basic algebra, Weight and Distance, Maximum likelihood decoding, Reliability of MLD, Error-detecting Codes, Error – correcting Codes. (Sections1.1-1.12 of Chapter 1).

**Learning outcomes**: Upon completion of this unit, the student will be able to: Understand the Effectsof error correction and Detection and the concept of Maximum-Likelihood Decoding and Reliability of MLD.

**UNIT – II**

**Linear Codes :** Linear Codes , Two important subspaces , Independence, Basis, Dimension,Matrices, Bases for C= <S> and $C^{\perp}$, Generating Matrices and Encoding . (Sections 2.1-2.6 of Chapter.2).

**Learning outcomes**: Upon completion of this unit, the student will be able to: Understand Basis,Dimension and GeneratingMatrices and Encoding.

**UNIT – III**

**Linear Codes :** Parity – Check Matrices, Equivalent Codes, Distance of a Linear Code, Cosets, MLD for Linear Codes, Reliability of IMLD for Linear Codes. (Sections 2.7-2.12 of Chapter.2).

**Learning outcomes**: Upon completion of this unit, the student will be able to: Understand Parity-Check Matrices, solving problems On linear codes.

**UNIT –IV**

**Perfect and Related Codes:** Some bounds for Code, Perfect Codes, Hamming Codes , Extended Codes, The extended Golay Code, Decoding the extended Golay Code, The Golay code, Reed – Mullar Codes, Fast decoding for RM (1,m).(Sections 3.1-3.9 of Chapter 3).

**Learning outcomes**: Upon completion of this unit, the student will be able to: Understand andimplement codes and source of information.

B. Sai

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**UNIT –V**

**Cyclic Linear Codes :** Polynomials and Words , Introduction to Cyclic codes, Polynomials encoding and decoding, Finding Cyclic Codes, Dual Cyclic Codes.(Sections 4.1-4.5 of Chapter 4).

**Learning outcomes:** Upon completion of this unit, the student will be able to: Understand Cycliccodes.

**PRESCRIBED BOOK: "CODING THEORY- THE ESSENTIALS"** , D.G. Hoffman, D.A. Lanonard , C.C. Lindner, K. T. Phelps,C. A. Rodger, J.R.Wall, Marcel Dekker Inc.

**REFERENCE BOOK: "Introduction to coding Theory"**, J.H. Van Lint, Springer Verlag.

**Course outcomes:** After completing this course, the student will be able to: learn some algebraicproperties of certain types of codes that at widely used in Engineering.

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CODE: M404(A)(21)(NR)

# M.Sc. DEGREE EXAMINATION, MARCH 202__

## FOURTH SEMESTER

## MATHEMATICS

## Paper –IV, ALGEBRAIC CODING THEORY

## Model paper

Time: Three hours Maximum : 70 marks

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Answer ONE question from each Unit (5 x 14 = 70 marks)

### UNIT-I

1 a) Construct the IMLD table for the code C ={ 101 ,111,011 } and verify whether this code C detects the error pattern u = 100.

b) Prove that a code C of distance d will at least detect all non zero error patterns of weight less than or equal to d-1 .Moreover, there is at least one error pattern of weight d which C will not detect.

(OR)

2 a) Define t – error detecting code. Find all error patterns which the code C = { 000 , 111 } will detect.

b) Let C = { 011 , 101, 110 }.Does C correct the error pattern u = 100 ?

### UNIT-II

3 a) Find the dual code $c^{\perp}$ for the code c=< $s$ >, when s= {1010, 0101, 1111}

b) construct examples in $k^5$of each of the following rules

1)u($v + w$)=u.v +u.w

2)a($v, w$)=($a. v$). $w$ = v.($a. w$)

(OR)

4 a) Define linearly independent set. verify whether the following set is independence. If the set is linearly dependent, extract from s a largest linearly ` independent sub, set whereS ={101, 011, 110,010}.

b)Find a basis for $c^{\perp}$ for the set s ={11101, 10110, 01011, 11010}

### UNIT-III

5 a) Find a parity check matrix for c ={000 ,001 ,010 ,011}

b)Define systematic code. Find a systematic equivalen to the given code C .check that C and $C^1$ have the same length dimension and distance where C ={00000 ,10110 ,1011 ,000111}

(OR)

6 a) Let H be a parity - cheek mate x for a linear code C Then show that C has distance d is and only of any set of d-1 rows of H is linearly independent and at least one set of d rows of H is linearly dependent.

b) construct an SDA assuming IMLD for the code C = {0000,1010,1101,0111}

B. Sol...

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UNIT-IV

7 a) Find an upper bound for the size or dimension of a linear code C of length n = 6 and distance d = 3.

b) Find a generator matrix for the standard form for a Hamming code of length 7 and encode the message 101.

(OR)

8 a) State and prove the Hamming Bound theorem.

b) Define (n,k,d) linear code. In any ( n, k ,d ) linear code, Prove that d=n-k+1 if and only if every n-k rows of the parity check matrix are linearly independent.

UNIT-V

9 a) Find a basics and generating matrix for the linear cyclic code of length n=7 and g(x) $=1+x^2+x^3$.

b) Let $g(x)=1+x^2+x^3$ be the generator polynomial of a linear cyclic code of length 7.

i)Encode the following message polynomial $1+ x^3$

ii)Find the message polynomial corresponding to the code word $x^2+x^4+x^5$ .

(OR)

10 a) If C is a linear cyclic code of length n and dimension k with generator g(x) anf if $1+x^n = g(x)h(x)$, then show that $C^{\perp}$ is a cyclic code of dimension n-k with generator $x^k.h(x^{-1})$

b) Find the generator polynomial for the dual code of the cyclic code of length n=6 having generator polynomial $g(x) =1+x^3$.

B. S

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**M.Sc. MATHEMATICS –IV SEMESTER**
**M 404B(21)(NR) – LATTICE THEORY**
**(With effect from the batch of students admitted during 2021-2022)**

| Subject Code : | M404(B)(21) | I A Marks | 30 |
|---|---|---|---|
| No. of Lecture / Seminar/ Tutorial for week | 06L + 01 S/T | End Exam Marks | 70 |
| | | Total Marks | 100 |

**Course Objectives :** To develop skills and knowledge of standard concepts in Hasse diagrams, completelattices, distributive lattices, Boolean algebras and Boolean ring.

**UNIT –I**
**Partly Ordered Sets:**
Set Theoretical Notations, Relations, Partly Ordered Sets, Diagrams, Special Subsets of a Partly OrderedSet, Length, Lower and Upper Bounds, The Jordan–Dedekind Chain Condition, Dimension Functions.(Sections 1-9 of Ch I)

**Learning outcomes:** Upon completion of this unit, the student will be able to: Understand partiallyordered sets and Jordan Dedekind chain conditions.

**UNIT – II**
Algebras, Lattices, The Lattice Theoretical Duality Principle, Semi Lattices, Lattices as Partly Ordered Sets, Diagrams of Lattices, Sub Lattices, Ideals, Bound Elements of a Lattice, Atoms and Dual Atoms, Complements, Relative Complements, Semi Complements, Irreducible Prime Elements of a Lattice, The Homomorphism of a Lattice, Axiom Systems of Lattices.(Sections 10-21 of Ch II)

**Learning outcomes:** Upon completion of this unit, the student will be able to: Analyze the relationship between posets and lattices, acquire knowledge of fundamental notions from latticetheory.

**UNIT – III**
Complete Lattices, Complete Sub Lattices of a Complete Lattice, Conditionally Complete Lattices,Compact Elements and Compactly Generated Lattices, SubAlgebra Lattice of an Algebra, Closure Operations, Galois Connections, Dedekind Cuts, Partly Ordered Sets as Topological Spaces.(Sections 22-29 of Ch III)

**Learning outcomes:** Upon completion of this unit, the student will be able to: Define and understandbasic properties of complete lattices and conditionally complete lattices, closure operations and their applications.

**UNIT – IV**
Distributive Lattices, Infinitely Distributive and Completely Distributive Lattices, Modular Lattices, Characterization of Modular and Distributive Lattices by their Sublattices, Distributive Sub lattices ofModular Lattices.(Sections 30-34 of Ch IV)

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**Learning outcomes:** Upon completion of this unit, the student will be able to: Characterize modularand distributive lattices using the Dedekind criterions.

**UNIT – V**

The Isomorphism Theorem of Modular Lattices, Covering Conditions, Meet Representation in Modular and Distributive Lattices.(Sections 35-36 of Ch IV) Boolean Algebras, De Morgan Formulae, Complete Boolean Algebras, Boolean Algebras and Boolean Rings.(Sections 42-46 of Ch VI)

**Learning outcomes:** Upon completion of this unit, the student will be able to: Characterize modularand distributive lattices using the Birkhoff and Understand Boolean algebras, Boolean rings.

**PRESCRIBED BOOK:** **"Introduction to Lattice Theory"**, Gabor Szasz, Acadamic press.

**REFERENCE BOOK:** **"Lattice Theory"**, G. Birkhoff, Amer. Math.Soc.

**Course outcomes:** After completing this course, the student shall be able to: attain some knowledge of basic concepts of structures with order relation, importance's of lattice and the relationship betweenBoolean algebras and Boolean Rings with unity.

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CODE: M404(B)(21)(NR)

M.Sc. DEGREE EXAMINATION, MARCH 202__

FOURTH SEMESTER

MATHEMATICS

Paper –IV, LATTICE THEORY

Model paper

Time: Three hours Maximum : 70 marks

Answer ONE question from each Unit (5 x 14 = 70 marks)

UNIT-I

1 a) State Zorn's Lemma and chain axiom.Show that the chain axiom implies Zorn's Lemma.

b) Prove that if every sub chain and every completely unordered subset of a poset P is finite the P is finite.

(OR)

2 a) Prove that a poset P satisfies the minimum condition if and only if every nonempty subset of P contains minimal elements.

b) Show that on a partially ordered set of locally finite length bounded below, the dimension function can be defined.

UNIT-II

3 a) Prove that every finite subset of the lattice L has an infimum and supremum.

b) Prove that two lattices are isomorphic if and only if they are order isomorphic.

(OR)

4 a) Prove that every uniquely complemented lattice is weakly complemented.

b) Prove that a lattice is a chain if and only if every one of its elements is meet irreducible.

UNIT-III

5 a) Define a complete lattice. Prove that every order preserving mapping of a complete lattice into itself has a fixed element .

b) Prove that every element of a lattice satisfying the maximum condition is compact.

(OR)

6 a) Give an example of a conditionally complete lattice which is not complete.

b) Prove that every lattice is isomorphic to some sublattice of a complete lattice.

UNIT-IV

7 a) Prove that every complete ring of sets is completely distributive.

b) Prove that a lattice L is modular if,and only if,every triplet a,b,c of L satisfies the equation $a \cup (b \cap (a \cup c))=(a \cup b) \cap (a \cup c)$.

(OR)

8 a) Prove that a lattice is distribution if and only if every one of triplets of elements has a median.

b) Prove that in a modular lattice ,the sub lattice generated by the elements x,y,z of the lattice is distributive if,and only if, $x \cap (y \cup z)=(x \cap y) \cup (x \cap z)$.

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## UNIT-V

9 a)Prove that transposed intervals of a modular lattice are isomorphic.
b)Prove that every elementof a distributive lattice has atmost one irredudant irreducible meet-representation.

(OR)

10 a) Show that the complemented elements of a bounded distributive lattice form a sublattice.
b)Prove that every complete Boolean algebra is infinitely distributive.

## M.Sc. MATHEMATICS –IV SEMESTER
## M 404C(21)(NR) – OPERATOR THEORY
**(With effect from the batch of students admitted during 2021-2022)**

| | | | |
|---|---|---|---|
| Subject Code : | M404(C)(21) | I A Marks | 30 |
| No. of Lecture / Seminar/ Tutorial for week | 06L + 01 S/T | End Exam Marks | 70 |
| | | Total Marks | 100 |

UNIT –I

Inner Product Space. Hilbert Space, Further Properties of Inner product Spaces, Orthogonal Complements and Direct Sums, Orthonormal sets and sequences, Series Related to Orthonormal sequences and sets. (Sections: 3.1 to 3.5 of Chapter 3)

UNIT – II

Total Orthonormal sets and sequences, Legendre, Hermite and Laguerre Polynomials, Representation of functionals on Hilbert Spaces, Hilbert-Adjoint Operator. (Sections: 3.6 to 3.9 of Chapter 3)

UNIT – III

Spectral theory in Finite Dimensional Normed Spaces, Basic Concepts, Spectral Properties of Bounded Linear Operators, Further Properties of Resolvent and Spectrum. (Sections: 7.1 to 7.4 of Chapter -7)

UNIT –IV

Banach Algebras, Further Properties of Banach Algebras, Compact Linear Operators on Normed spaces, Further Properties of Compact Linear Operators, Spectral Properties of Compact Linear Operators on Normed Spaces. (Sections: 7.6 to 7.7 of Chapter 7 and Sections 8.1 to 8.3 of Chapter -8)

UNIT –V

Further Spectral properties of Compact Linear Operators, Operator Equations Involving Compact Linear Operators, Further Theorems of Fredholm type, Fredholm alternative. (Sections: 8.4 to 8.7 of Chapter -8)

TEXT BOOK:

INTRODUCTORY FUNCTIONAL ANALYSIS WITH APPLICATIONS: **Erwin Kreyszig**, John Wiley & Sons.

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CODE: M404(C)(21)(NR)

M.Sc. DEGREE EXAMINATION, MARCH 202__

FOURTH SEMESTER

MATHEMATICS

Paper –IV, OPERATOR THEORY

Model paper

Time: Three hours Maximum : 70 marks

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Answer ONE question from each Unit (5 x 14 = 70 marks)

UNIT – 1

1 (a) State and prove Schwarz inequality.

(b) Let Y be any subspace of a Hilbert space H. Then prove that $H = Y \oplus Z$, where $Z = Y^{\perp}$

(OR)

2. (a) Let X be an inner product space and $M \neq \phi$ a convex subset which is complete in the metric induced by the inner product. Then prove that for every given x in X there exists a unique y in M such that $\delta = \inf_{\tilde{y} \in M} \left\| x - \tilde{y} \right\| = \| x - y \|$.

(b) Describe the method of Gram – Schmidt process for orthonormalizing a linearly independent sequence in an Inner product space.

**UNIT II**

3 (a) Let M be subset of an inner product space X. If M is total in X, then prove that there does not exist a nonzero x in X which is orthogonal to every element of M; i.e., $x \perp M \Rightarrow x = 0$.

(b) Let H be a Hilbert space. If H is separable then prove that every orthonormal set in H is countable.

(OR)

4. State and prove Riesz theorem of functional on Hilbert spaces.

**UNIT III**

5 (a) Give an example for an operator with a spectral value which is not an Eigen value.

(b) Let $T \in B(X,X)$, where X is a Banach space. Prove that if $\|T\| < 1$ then $(I - T)^{-1}$ exists as a bounded linear operator on the whole space X and

$$(I - T)^{-1} = \sum_{j=0}^{\infty} T^j = I + T + T^2 + ......$$

(OR)

6 State and prove spectral mapping theorem for polynomials.

## UNIT IV

7. Let A be a complex Banach algebra with identity e. Then prove that for any $x \in A$, $\sigma(x) \neq \emptyset$.

(OR)

8 . (a) Let T: X → X be a compact linear operator on a normed space X. Then prove that for every $\lambda \neq 0$, the range of $T_\lambda = T - \lambda I$ is closed.

(b) Let X and Y be normed spaces. Let T: X → Y be a linear operator. Then prove that T is compact if and only if it maps every bounded sequence $\{x_n\}$ in X onto a sequence $\{Tx_n\}$ in Y which has a convergent subsequence.

## UNIT V

9. (a) Let T: X → X be a compact linear operator on a normed space X, and let $\lambda \neq 0$. Prove that there exists a smallest integer q (depending on $\lambda$ ) such that from n = q on, the ranges $T_\lambda^n(X)$ are all equal; and if q > 0, the inclusions

$T_\lambda^0(X) \supset T_\lambda(X) \supset ...... \supset T_\lambda^q(X)$ are all proper.

(b) Let T: X → X be a compact linear operator on a normed space X. Prove that if T has nonzero spectral values, then every one of them must be an Eigen value of T .

(OR)

10. Let J = [a,b] be any compact interval and suppose that k is continuous on J × J. Then prove that the operator T: X→X defined by $(Tx)(s) = \int_a^b k(s,t)x(t)dt$ , where X = C[a,b], is a compact linear operator.

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## M.Sc. MATHEMATICS-IV SEMESTER

## M405 (A)(21): COMMUTATIVE ALGEBRA

### (With effect from the batch of students admitted during 2021-2022)

| | | | |
|---|---|---|---|
| Subject Code : | M405(A)(21) | I A Marks | 30 |
| No. of Lecture / Seminar/ Tutorial for week | 06L + 01 S/T | End Exam Marks | 70 |
| | | Total Marks | 100 |

**Course Objectives:**

To introduce the studentsvariousconcepts of integral domains, valuation rings, primary decomposition in Noetherian rings, Arin rings, Dedekind domains and fractional ideals.

**UNIT I**

Integral dependence- the going-up theorem-Integrally closed integral domains.

**Learning outcomes:** Upon completion of this unit, the student will be able to: understand integral dependence, the going-up theorem and integrally closed integral domains.

**UNIT II**

The going-down theorem - valuation rigs.

**Learning outcomes:** Upon completion of this unit, the student will be able to: understand the going-down theorem and valuation rings and apply them.

**UNIT III**

Chain conditions

**Learning outcomes:** Upon completion of this unit, the student will be able to: know and understand different kinds of chain conditions and their role in the structures of different kinds of rings.

**UNIT IV**

Noetherian rings: Primary decomposition of Noetherian rings - Artin rings.

**Learning outcomes:** Upon completion of this unit, the student will be able to: understand primary decomposition in Noetherian rings and structures of Artin rings.

**UNIT V**

Discrete valuation rings - Dedekind domains -Fractional ideals.

**Learning outcomes:** Upon completion of this unit, the student will be able to: understand Discrete valuation rings, Dedekind domains and Fractional ideals.

**TEXT BOOK:**

Introduction to commutative algebra by M.F.Atiya and I.G. Macdonald, Addison-Welsey Publishing Company.

**Reference Books:**

1. T.W Hungerford, *Algebra,* Springer-Verlag
2. N.S Gopalakrishan, *Commutative Algebra,* Oxonian Press

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CODE: M405(A)(21)

M.Sc. DEGREE EXAMINATION,
FOURTH SEMESTER
MATHEMATICS
Paper –V, COMMUTATIVE ALGEBRA
Model paper

Time: Three hours — Maximum : 70 marks

Answer ONE question from each Unit — (5 x 14 = 70 marks)

UNIT I

1. (a) Let B be a commutative ring with an identity element and A be a subring of B. Then prove that the following are equivalent:
   (i) $x \in B$ is integral over A;
   (ii) A[x] is a finitely generated A-module;
   (iii) A[x] is contained in a subring C of B such that C is a finitely generated A-module.

   (b) Let $A \subseteq B$ be integral domains and B integral over A. The show that B is a field if and only if A is a field.

Or

2. (a) State and prove the going-up theorem.

   (b) Let $A \subseteq B$ be commutative rings with an identity elements and C the integral closure of A in B. Let S be a multiplicatively closed subset of A. The prove that $S^{-1}C$ is the integral closure of $S^{-1}A$ in $S^{-1}B$.

UNIT II

3. (a) State and prove the going-down theorem.

   (b) Let A be an integrally closed domain, K its field of fractions and L a finite separable algebraic extension of K and B be the integral closure of A in L. The prove that there exists a basis $v_1, v_2, \dots, v_n$ of L over K such that $B \subseteq Av_1 + Av_2 + \dots + Av_n$.

Or

4. (a) Let K be a field and L be an algebraically closed field. Let $\Sigma$ be the poset of all pairs (A, f), where A is a subring of K and f is a homomorphism of A into L under natural ordering. Let (B, g) a maximal element of $\Sigma$. Then prove that B is a valuation ring of the field K.

   (b) Let A be a subring of a field K. The prove that the integral closure of A in K is

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the intersection of all valuation rings of K which contain A.

UNIT III

5. (a) Prove that M is a Noetherian A-module if and only if every submodule of M is finitely generated.

(b) If $M_i$, i = 1, 2, 3, … , n, are Noetherian A-modules then prove that their direct product is also Noetherian.

Or

6. (a) Suppose that a module M has a composition series of length n. Then prove that every composition series of M has length n and every chain in M can be extended to a composition series.

(b) Prove that a module M has a composition series if and only in M satisfies both the chain conditions.

UNIT IV

7. (a) State and prove Hilbert's basis theorem.

(b) Prove that in a Noetherian ring every irreducible ideal is primary.

.Or

8 (a) Prove that in an Artin ring the nilradical is nilpotent.

(b) State and prove structure theorem for Artin rings.

UNIT V

9. (a)Define a discrete valuation ring and give an example. Let A be a Noetherian local domain of dimension one, M its maximal ideal and K= A/M its residue field. Then prove that the following are equivalent:

(i) A is a discrete valuation ring;
(ii) A is integrally closed;
(iii) M is a principal ideal.

(b) Prove that the ring of integers in an algebraic number field K is a Dedekind domain.

Or

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10. (a ) Define a fractional ideal. Show that a local domain A is a discrete valuation ring if and only if every non-zero fractional ideal of A is invertible.

(b) Prove that an integral domain A is a Dedekind domain if and only if every non-zero fractional ideal is invertible.

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M 405 (C) (21) NR

# M.Sc. MATHEMATICS –IV SEMESTER
## M 405(B)(21)(NR) – OPERATIONS RESEARCH
### (With effect from the batch of students admitted during 2021-2022)

| | | | |
|---|---|---|---|
| Subject Code : | M405(B)(21) | I A Marks | 30 |
| No. of Lecture / Seminar/ Tutorial for week | 06L + 01 S/T | End Exam Marks | 70 |
| | | Total Marks | 100 |

**Course Objectives:** To develop problem solving skills of linear programming problems using Two- Phase method, Duality theory, The Revised Simplex method, Game theory and integer Programming

**UNIT –I**

**Further Discussion of the simplex method:** Further discussion; the two phase Method for artificial variables; phase-I; Phase-II; Numerical examples of the two phase method. [Sections 5.1 to 5.4 of Chapter -5 of [1] ]

**Learning outcome:** Upon completion of this unit, the student will be able to: Solve the LPP using the two phase method.

**UNIT –II**

**Duality theory and its Ramifications:** Alternative formulations of linear programming problems; Dual linear programming problems; Fundamental properties of dual problems; other formulations of dual problems; unbounded solution in the primal; the dual simplex algorithm –an example. Post optimality problems, changing the price vector, changing the requirements vector, adding variables or constraints
(Sections 8.1 to 8.7; 8.10 of Chapter 8 and 11.2 to 11.5 Chapter 11 of [1] ).

**Learning outcome**: Upon completion of this unit, the student will be able to: Find the dual of an LPP and solve the Problem.

**UNIT –III**

**The Revised simplex method:** Introduction; Revised simplex method-standard form I; computational procedure for standard form I; Revised simplex method-Standard form II; computational procedure for standard form II; Initial identity matrix for phase –I ; comparison of the simplex method and Revised simplex method. (Sections 7.1 to 7.6 & 7.8 of Chapter 7 of [1]).

**Learning outcome**: Upon completion of this unit, the student will be able to: Solve a linear programming problem using Revised Simplex Method.

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**UNIT –IV**

**Game theory:** Game theory and Linear programming; Introduction; reduction of a game to a linear programming problem; conversion of a linear programming problem to a game problem.

(Sections 11.2 to 11.14 of Chapter 11 of [1] )

**Learning outcome**: Upon completion of this unit, the student will be able to: Solve Linear programming problems and game theory problems.

**UNIT –V**

**Goal programming, Integer programming:** Introduction; Gomory's cut, Balas Implicit Enumeration Technique, Goal programming.

(Sections 7.1, 7.2 and 7.4 of Chapter 7 and Section 10.3 of Chapter10 of [2] )

**Learning outcome**: Upon completion of this unit, the student will be able to: Understand and Solve Goal programming problems and integer programming problems.

**TEXT BOOKS:**

[1] **G.Hadley** "Linear programming" Addison Wesley Publishing Company.
[2] **Benjamin Lev and Howard J. Weiss** "Introduction to Mathematical Programming" Edward Arnold Pub, London, 1982.

**Course outcomes:** After completing this course, the student shall learn in detail, about the Simplex method, Revised simplex method and the relation between Game theory and linear programming.

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CODE: M405(C)(21)(NR)

M.Sc. DEGREE EXAMINATION, MARCH 202__

FOURTH SEMESTER

MATHEMATICS

Paper - V, OPERATIONS RESEARCH

Model paper

Time: Three hours | Maximum : 70 marks

Answer ONE question from each Unit (5 x 14 = 70 marks)

UNIT I

1. Use two – phase simplex method to solve

Maximize $Z = 5x_1 - 4x_2 + 3x_3$

Subject to $2x_1 + x_2 - 6x_3 = 20$ ;

$6x_1 + 5x_2 + 10x_3 \leq 76$

$8x_1 - 3x_2 - 6x_3 \leq 50$, $x_1, x_2, x_3 \geq 0$.

(OR)

2. Use two – phase method to solve

Minimize $Z = 5x_1 + 8x_2$

Subject to $3x_1 + 2x_2 \geq 3$

$x_1 + 4x_2 \geq 4$

$x_1 + x_2 \leq 5$ and $x_1, x_2 \geq 0$.

UNIT II

3. Use duality to solve

Maximize $Z = 2x_1 + x_2$

Subject to $x_1 + 2x_2 \leq 10$,

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$x_1 + x_2 \le 6$

$x_1 - x_2 \le 2$

$x_1 - 2x_2 \le 1,\ x_1,\ x_2, \ge 0.$

**(OR)**

4. **Use dual Simplex method to solve**

**Maximize** $Z = 10x_1 + 6x_2 + 2x_3$

**Subject to** $-x_1 + x_2 + x_3 \ge 1,$

$3x_1 + x_2 - x_3 \ge 2,\ x_1,\ x_2,\ x_3 \ge 0.$

**UNIT III**

5. **Describe the major steps involved in Revised Simplex method to solve an L.P.P.**

**(OR)**

6 **Use revised simplex method to solve the L.P.P.**

**Maximize** $Z = x_1 + 2x_2$

**Subject to** $x_1 + x_2 \le 3,$

$x_1 + 2x_2 \le 5$

$3x_1 + x_2 \le 6$ **and** $x_1,\ x_2 \ge 0.$

**UNIT IV**

7. **Explain the conversion of a linear programming problem to a game problem.**

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**(OR)**

8. Solve the following game for the pay-off matrix

| | | Player B | | |
|---|---|---|---|---|
| | | $B_1$ | $B_2$ | $B_3$ |
| | $A_1$ | 1 | 3 | 1 |
| Player A | $A_2$ | 0 | -4 | -3 |
| | $A_3$ | 1 | 5 | -1 |

**UNIT V**

9. Solve the following goal programming

$$\text{Min } Z = P_1 d_1^- + P_2 d_2^- + 2P_3 d_3^- + P_3 d_1^+$$

Subject to $10x_1 + 10x_2 + d_1^- - d_1^+ = 400$

$x_1 + d_2^- = 40$

$x_2 + d_3^- = 30$, $x_1, x_2, d_1^-, d_1^+, d_2^-, d_3^- \geq 0$

**(OR)**

10. Use Gomorian fractional cut method to solve the following L.P.P

Maximize $Z = x_1 + 2x_2$

Subject to $x_1 + 2x_2 \leq 12$

$4x_1 + 3x_2 \leq 14,$

$x_1, x_2 \geq 0$ and are integers.

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**M.Sc. MATHEMATICS-IV SEMESTER**

**M405(C)(21)(NR): BANACH ALGEBRAS**

**(With effect from the batch of students admitted during 2021-2022)**

| | | | |
|---|---|---|---|
| Subject Code : | M405(C)(21) | I A Marks | 30 |
| No. of Lecture / Seminar/ Tutorial for week | 06L + 01 S/T | End Exam Marks | 70 |
| | | Total Marks | 100 |

**Course Objectives:**To introduce the students Banach algebraswhich includes its basic concepts and the spectrum, the radical, the structure of commutative Banach Algebras, and involutions and some special commutative Banach algebras.

**UNIT-I**

General preliminaries on Banach Algebras: The definition and examples – Regular and singular elements – Topological divisors of Zero.

**Learning outcomes:** Upon completion of this unit, the student will be able to: know basic notions in Banach algebras and examples of Banach algebras and understand regular and singular elements and topological divisors of zero in Banach algebras.

**UNIT-II**

The spectrum -The formula for the spectral radius.

**Learning outcomes:** Upon completion of this unit, the student will be able to: understand the spectrum spectral formula in Banach spaces.

**UNIT-III**

The radical and the semi – simplicity - The structure of commutative Banach Algebras: The Gelfand mapping.

**Learning outcomes:** Upon completion of this unit, the student will be able to: understand the radical and semi-simplicity of Banach algebras and the structure of the commutative Banach algebras.

**UNIT-IV**

Applications of the formula $r(x) = \lim\|x^n\|^{1/n}$ - Involutions in Banach Algebras: The Gelfand–Neumark theorem.

**Learning outcomes:** Upon completion of this unit, the student will be able to: apply spectral formula and understand involutions in Banach algebras.

**UNIT-V**

Some special commutative Banach Algebras: Ideals in C(x) and the Banach–Stone theorem The stone–Cech compactification – commutative C* - algebras.

**Learning outcomes:**Upon completion of this unit, the student will be able to: know and understand some special commutative Banach algebras.

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**Text Book:** Introduction to Topology and Modern Analysis – By G.F. Simmons – Tata McGraw – Hill Edition, 2004.

**Reference Books:**

1. E. Kaniuth, A Course in Commutative Banach Algebras, Springer, New York, 2009.
2. R. Larsen, Banach Algebras, Marcell-Dekker, 1973.
3. Banach Algebras and Automatic Continuity, London Mathematical Society, Monographs, 2001.

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B. Sat

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CODE: M405(~~C~~ B)(21)

# M.Sc. DEGREE EXAMINATION,
## FOURTH SEMESTER
## MATHEMATICS
## Paper –V, BANACH ALGEBRAS
### Model paper

Time: Three hours  Maximum : 70 marks

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Answer ONE question from each Unit  (5 x 14 = 70 marks)

### UNIT I

1. (a) Let A be a Banach algebra. Then prove that for every element x in A for which $\|1 - x\| < 1$ is regular and the inverse of such an element is given by $x^{-1} = 1 + \sum_{n=1}^{\infty}(1-x)^n$.

(b) Let A be a Banach Algebra and G be the set of regular elements of A. Then show that the mapping $x \to x^{-1}$ of G into G is continuous and is a homeomorphism of G onto itself.

Or

2. (a) Let A be a Banach algebra and S be the set of singular elements of A. The Prove that S is a closed subset of A.

(b) Let A be a Banach algebra and S be the set of singular elements of A and Z is the set of topological divisors of zero in A. The prove that the boundary of S is a subset of Z.

### UNIT II

3. (a) Let A be a Banach algebra and $x \in A$. Then prove that $\sigma(x)$ is non-empty.

(b) Let A be a Banach subalgebra of a Banach algebra B and $x \in A$. Then prove that $\sigma_B(x) \subseteq \sigma_A(x)$ and each boundary point of $\sigma_A(x)$ is a boundary point of $\sigma_B(x)$.

Or

4. Let A be a Banach algebra and $x \in A$. Define the spectral radius r(x) of x and prove that $r(x) = \lim \| x^n \|^{1/n}$.

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UNIT III

5. (a) If R is the radical of a Banach algebra A and $r \in R$, the prove that $1 - r$ is regular.

(b) If r is an element of a Banach algebra A with the property that $1 - xr$ is regular for every x in A then prove that r is in R, where R is the radical of A.

Or

6. (a) If $f_1$ and $f_2$ are multiplicative functionals on a commutative Banach algebra A with same null space M then prove that $f_1 = f_2$.

(b) Prove that $M \to f_M$ is a one-to-one mapping of the set of all maximal ideals of a commutative Banach algebra A onto the set of all its multiplicative functionals.

UNIT IV

7. (a) Define Banach $^*$-algebra and $B^*$-algebra and give one example for each of These.

(b) Define a normal element in a $B^*$-algebra. If x is a normal element in a $B^*$-algebra then show that $\|x^2\| = \|x\|^2$.

.Or

8 State and prove the Gelfand-Neumark theorem.

.UNIT V

9. State and prove Banach-Stone theorem.

Or

10. (a) Let A be a commutative $C^*$-algebra of operators on a non-trivial Hilbert space H. If an operator in A is regular in B(H), then prove that it is also regular in A.

(b) Let N be a normal operator on a non-trivial Hilbert space H, A the commutative $C^*$-algebragenerated by N, and the space of maximal ideals in A. Then prove that the function $N^{\wedge}$ in C( ) which corresponds to N under the Gelfand mapping is a homeomorphism of onto σ(N).

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B. S...
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P.G. Board of Studies in Mathematics
Acharya Nagarjuna University
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